The Layer Model

of the Greenhouse Effect

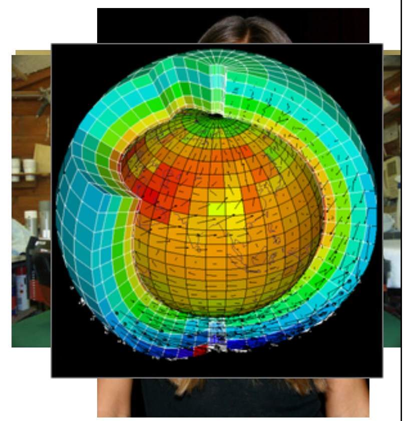
ATS 150

Lecture 4

Please read Chapter 3
in Archer Textbook

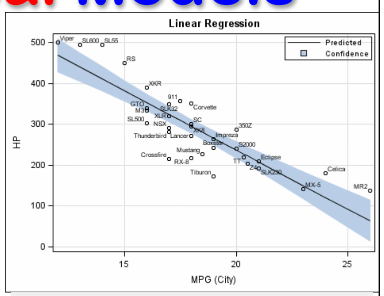
What is a Model?

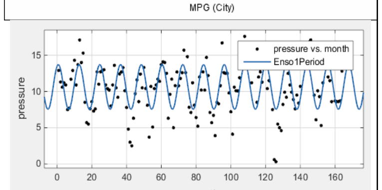
- Models represent reality in a simplified or idealized way
- Used to understand or predict
- Doesn't have to be realistic to be useful



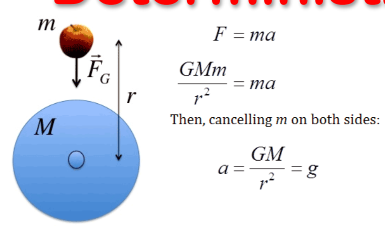
Empirical Models

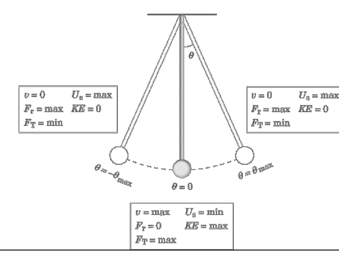
- Generalized mathematical formulation with adjustable coefficients
- Combinations of
  - polynomials
  - exponential growth & decay
  - Periodic sines and cosines
- Coefficients fit to data (e.g., least squares)
- Interpolation or extrapolation





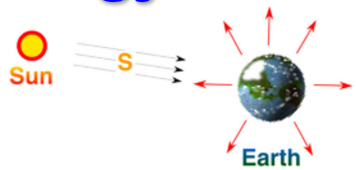
Deterministic Models





- Formulated as "cause and effect"
- Common in physics and chemistry
- Usually take the form of differential equations
- Initial & boundary-value problems
- May still have adjustable coefficients

Energy Balance



Energy In = Energy Out

- **Let the rate of energy flow from the Sun to the Earth be called F_{in}**
- **Let the rate of energy flow from the Earth to outer space be called F_{out}**

Energy Balance

$F_{in} = F_{out}$

- **Assume both Sun and Earth are blackbodies (ε=1), so $F = \sigma T^4$**
- **F_{in} = absorbed sunlight x daylight area**
- **$F_{out} = \sigma T_{earth}^4$ x total area**

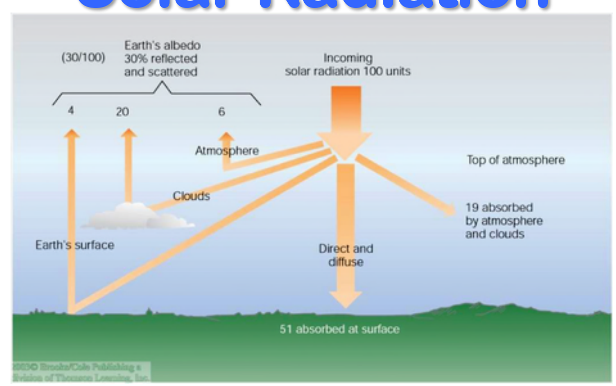
Energy Balance

$F_{in} = F_{out}$

F_{in} = Solar brightness x (1 – albedo)
x (area of Earth's shadow)

$F_{out} = \sigma (T_{earth})^4$ x (area of Earth's surface)

Solar Radiation



- **30% reflected by clouds, air, dust, and surface**
- **19% absorbed by the atmosphere (mostly clouds)**
- **51% absorbed at the surface**

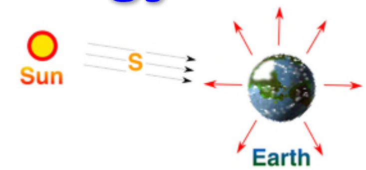
Energy Balance

$F_{in} = F_{out}$

F_{in} = Solar brightness x $(1 - \alpha)$ x πr^2

$F_{out} = \sigma (T_e)^4 \times 4\pi r^2$

Energy Balance



$F_{in} = F_{out}$

$$S(1-\alpha)\pi r^2 = 4\pi r^2 \sigma T^4$$

$$S(1-\alpha) = 4\sigma T^4$$

$$\frac{S(1-\alpha)}{4\sigma} = T^4$$

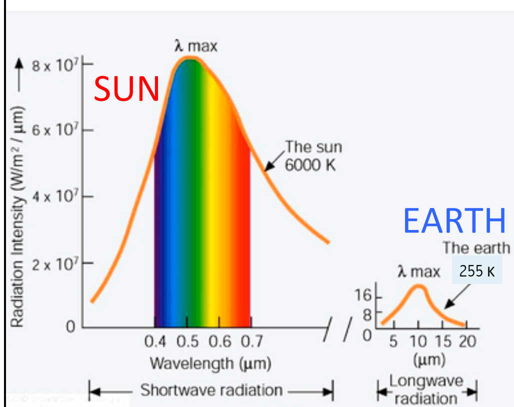
Energy Balance

$F_{in} = F_{out}$

$$T = \left(\frac{S(1-\alpha)}{4\sigma}\right)^{1/4} = \left(\frac{(1360\,Wm^{-2})(1-0.3)}{4(5.67x10^{-8}\,Wm^{-2}K^{-4})}\right)^{1/4} = 255K$$

But the observed
surface temperature is about 288 K

Thermal Emission

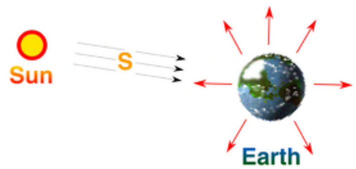


Both are almost perfect blackbodies!

The hot sun radiates at shorter (visible) wavelengths that carry more energy

Energy absorbed by the cooler earth is then re-radiated at longer (thermal infrared) wavelengths

Works for Other Planets Too

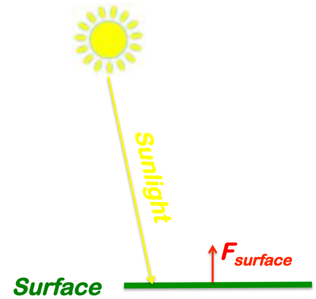


$F_{in} = F_{out}$

$$\frac{S(1-\alpha)}{4\sigma} = T^4$$

Experiment! http://tinyurl.com/planetary-balance

Bare Rock Model

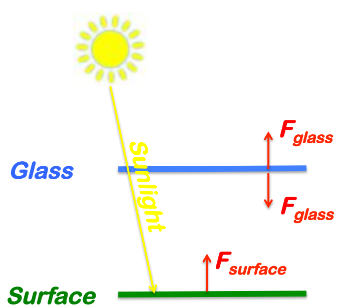


$F_{in} = F_{out}$

$$\frac{S(1-\alpha)}{4} = \sigma T^4$$

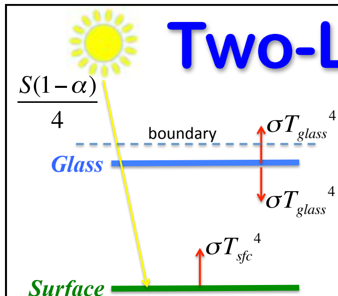
- If sunlight were the only source of energy, the surface of the Earth would be way too cold for life!
- 255 °K = -18 °C = -1 °F

Two-Layer Model



- Now imagine a layer of glass above the solid rock
- Glass is transparent to sunlight
- Glass is opaque to "Earthlight" (thermal IR)

Two-Layer Model



$F_{in} = F_{out}$

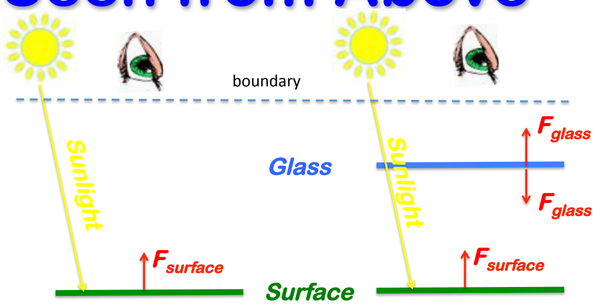
Boundary $\frac{S(1-\alpha)}{4} = \sigma T_{glass}^4$

Glass $\sigma T_{sfc}^4 = 2\sigma T_{glass}^4$

Surface $\frac{S(1-\alpha)}{4} + \sigma T_{glass}^4 = \sigma T_{sfc}^4$

- Sun shines right through transparent glass layer
- Glass layer emits both upward and downward
- Glass layer "feels" both space and surface
- Surface "feels" both Sun and glass

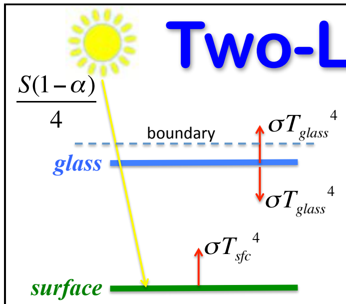
Seen from Above



Both models:

- Absorb the same sunshine
- Reflect the same amount of sunlight back out
- Emit the same amount of Earthlight

Two-Layer Model



$F_{in} = F_{out}$

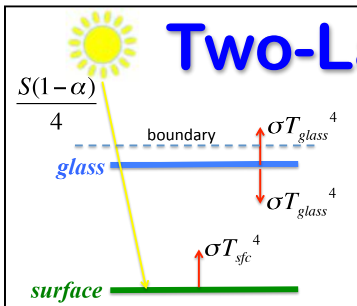
Boundary $\frac{S(1-\alpha)}{4} = \sigma T_{glass}^4$

Glass $\left(\frac{S(1-\alpha)}{4\sigma}\right)^{1/4} = T_{glass}$

$255\text{ K} = T_{glass}$

- Use balance at boundary to space to find temperature of the glass
- Since energy in from Sun and energy out from glass are same as bare rock model, temperature is too!

Two-Layer Model



$F_{in} = F_{out}$

Surface $\frac{S(1-\alpha)}{4} + \sigma T_{glass}^4 = \sigma T_{sfc}^4$

$$\cancel{\sigma}T_{glass}^4 + \cancel{\sigma}T_{glass}^4 = \cancel{\sigma}T_{sfc}^4$$

$$2T_{glass}^4 = T_{sfc}^4$$

$$\sqrt[4]{2}T_{glass} = T_{sfc} = 1.189\,T_{glass} = 303\,K$$

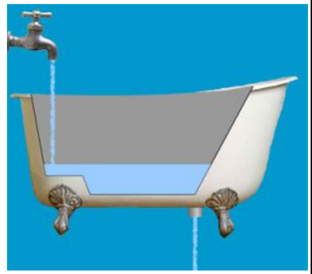
- Recall from boundary balance that
  $S(1-\alpha)/4 = \sigma T_{glass}^4$
- Substitute this into surface balance
- Find that surface is much warmer under glass!
- 303 °K = 30 °C = 86 °F

Summary

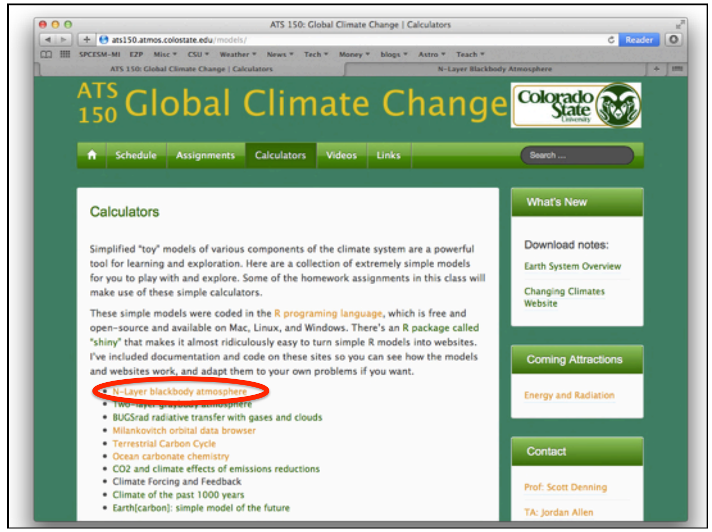
- Layer models are not meant to be accurate or predictive, just to help us understand how the world works
- Energy balance of "bare rock" model is way too cold to support life!
- Adding a layer of glass makes the surface nice and toasty
- Where did the heat come from?

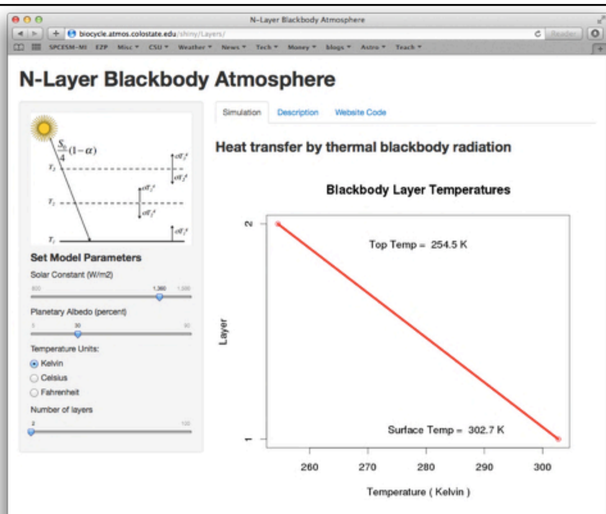
Bathtub Analogy

- If faucet runs faster than drain, level rises
- And vice versa
- Drain runs faster when water is deep
- Adding glass to layer model acts like a clog in the drain
- Water rises until drainage = inflow again



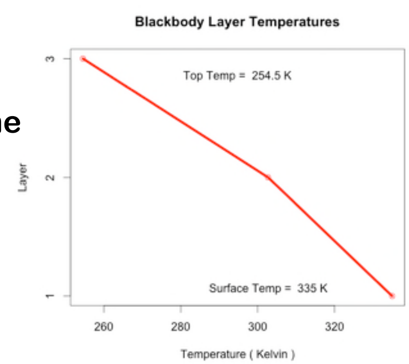
Faucet ~ Sun
Drain ~ Thermal emission
Water level ~ temperature





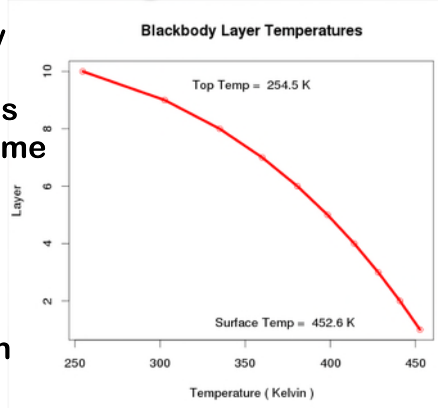
Three Layers

- Top boundary ("skin") temperature is always the same
- As we add layers, the surface gets hotter
- (bathtub drain gets slower)



Ten Layers

- Top boundary ("skin") temperature is always the same
- As we add layers, the surface gets hotter
- (bathtub drain gets slower)



100 Layers!

- Top boundary ("skin") temperature is always the same
- As we add layers, the surface gets hotter
- (bathtub drain gets slower)