The Layer Model of the Greenhouse Effect

ATS 150
Lecture 4

Please read Chapter 3 in Archer Textbook

Empirical Models

- Generalized mathematical formulation with adjustable coefficients
- Combinations of polynomials
- exponential growth & decay
- Periodic sines and cosines
- Coefficients fit to data (e.g., least squares)
- Interpolation or extrapolation

Deterministic Models

- Formulated as “cause and effect”
- Common in physics and chemistry
- Usually take the form of differential equations
- Initial & boundary-value problems
- May still have adjustable coefficients

What is a Model?

- Models represent reality in a simplified or idealized way
- Used to understand or predict
- Doesn’t have to be realistic to be useful

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Energy Balance

- Let the rate of energy flow from the Sun to the Earth be called $F_{in}$
- Let the rate of energy flow from the Earth to outer space be called $F_{out}$

Energy In = Energy Out

$F_{in} =\text{Solar brightness }\times (1 - \text{albedo}) \times \text{area of Earth’s shadow}$

$F_{out} = \sigma (T_{\text{earth}})^4 \times \text{area of Earth’s surface}$

Energy Balance

- Assume both Sun and Earth are blackbodies ($\varepsilon=1$), so $F = \sigma T^4$
- $F_{in} = \text{absorbed sunlight }\times \text{daylight area}$
- $F_{out} = \sigma T_{\text{earth}}^4 \times \text{total area}$

Solar Radiation

- 30% reflected by clouds, air, dust, and surface
- 19% absorbed by the atmosphere (mostly clouds)
- 51% absorbed at the surface
Energy Balance

\[ F_{\text{in}} = F_{\text{out}} \]

\[ F_{\text{in}} = \text{Solar brightness} \times (1 - \alpha) \times \pi r^2 \]

\[ F_{\text{out}} = \sigma (T_e)^4 \times 4\pi r^2 \]

Energy Balance

\[ F_{\text{in}} = F_{\text{out}} \]

\[ S(1-\alpha)\pi r^2 = 4\pi r^2 \sigma T^4 \]

\[ S(1-\alpha) = 4\sigma T^4 \]

\[ \frac{S(1-\alpha)}{4\sigma} = T^4 \]

But the observed surface temperature is about 288 K

Thermal Emission

Both are almost perfect blackbodies!

The hot sun radiates at shorter (visible) wavelengths that carry more energy.

Energy absorbed by the cooler earth is then re-radiated at longer (thermral infrared) wavelengths.
**Works for Other Planets Too**

\[ F_{\text{in}} = F_{\text{out}} \]

\[ \frac{S(1 - \alpha)}{4\sigma} = T^4 \]

Experiment! [http://tinyurl.com/planetary-balance](http://tinyurl.com/planetary-balance)

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**Bare Rock Model**

- If sunlight were the only source of energy, the surface of the Earth would be way too cold for life!
- \( 255 \, ^\circ K = -18 \, ^\circ C = -1 \, ^\circ F \)

\[ F_{\text{in}} = F_{\text{out}} \]

\[ \frac{S(1 - \alpha)}{4} = \sigma T^4 \]

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**Two-Layer Model**

- Now imagine a layer of glass above the solid rock
- Glass is transparent to sunlight
- Glass is opaque to “Earthlight” (thermal IR)

\[ \text{Glass} \]

\[ \text{Surface} \]

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**Two-Layer Model**

- Sun shines right through transparent glass layer
- Glass layer emits both upward and downward
- Glass layer “feels” both space and surface
- Surface “feels” both Sun and glass

\[ \text{Glass} \]

\[ \text{Surface} \]
Seen from Above

Both models:
• Absorb the same sunshine
• Reflect the same amount of sunlight back out
• Emit the same amount of Earthlight

Two-Layer Model

- Use balance at boundary to space to find temperature of the glass
- Since energy in from Sun and energy out from glass are same as bare rock model, temperature is too!

Two-Layer Model

- Recall from boundary balance that
  \( \frac{S(1-\alpha)}{4} = \sigma T_{\text{glass}}^4 \)
- Substitute this into surface balance
- Find that surface is much warmer under glass!
  \( \frac{S(1-\alpha)}{4} + \sigma T_{\text{glass}}^4 = \sigma T_{\text{glc}}^4 \)
  \( \frac{\sqrt{T_{\text{glass}}^4}}{\sqrt{2}} = T_{\text{glc}} = 1.189 T_{\text{glass}} = 303 \, \text{K} \)
  \( 2T_{\text{glass}} - T_{\text{glc}} = 86 \, ^\circ\text{F} \)

Summary

- Layer models are not meant to be accurate or predictive, just to help us understand how the world works
- Energy balance of “bare rock” model is way too cold to support life!
- Adding a layer of glass makes the surface nice and toasty
- Where did the heat come from?

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Bathtub Analogy

- If faucet runs faster than drain, level rises
- And vice versa
- Drain runs faster when water is deep
- Adding glass to layer model acts like a clog in the drain
- Water rises until drainage = inflow again

Faucet ~ Sun
Drain ~ Thermal emission
Water level ~ temperature

Three Layers

- Top boundary ("skin") temperature is always the same
- As we add layers, the surface gets hotter
- (bathtub drain gets slower)
Ten Layers

- Top boundary (“skin”) temperature is always the same
- As we add layers, the surface gets hotter
- (bathtub drain gets slower)

100 Layers!

- Top boundary (“skin”) temperature is always the same
- As we add layers, the surface gets hotter
- (bathtub drain gets slower)