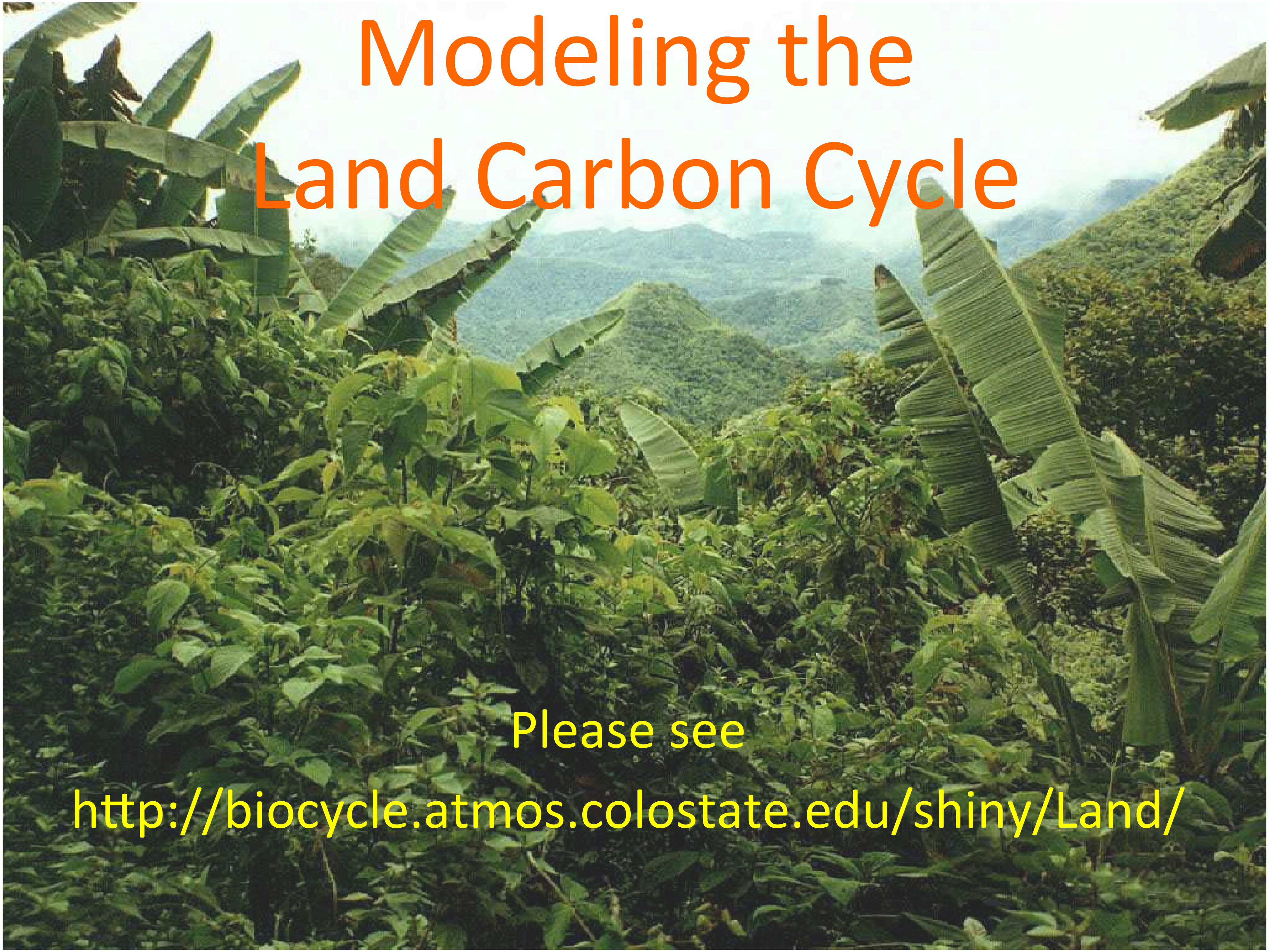
# Modeling the Land Carbon Cycle



Please see

http://biocycle.atmos.colostate.edu/shiny/Land/

# What is a Model?

- Models are simplifications of reality which we use to organize and make sense of what we see in the world around us
- Models are abstractions which seek to capture the essence of real processes
- Simple models are not intended to be perfect representations of the real world
- Models help us understand

# Levels of Abstraction

More abstract

6. Computer program
   (Stella, MATLAB, Python, R)

5. Numerical algorithm
   (arithmetic procedure)

4. Discrete model (algebra)

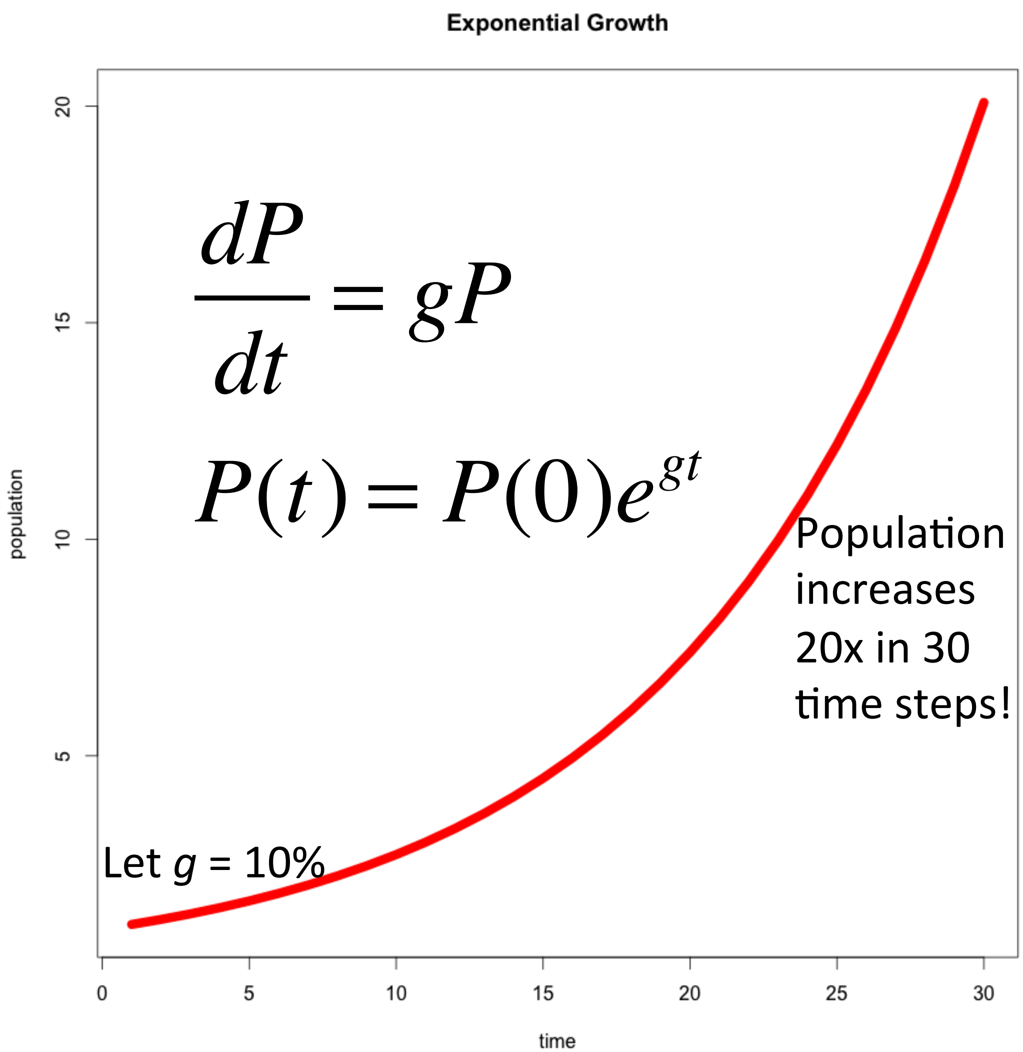
3. Mathematical model
   (differential equations)

2. Conceptual model
   (in our heads,
   maybe with a diagram)

1. Real world
   (impossibly complicated)

More realistic

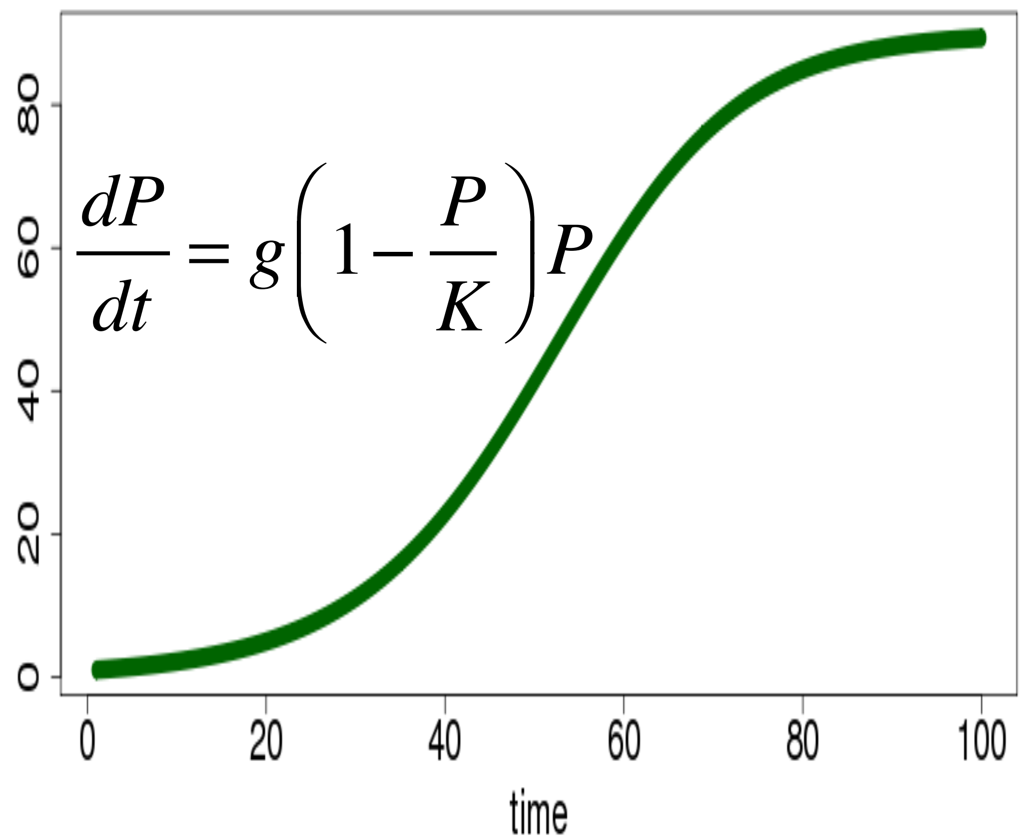
# Exponential Growth




- Rate of growth of new stuff (plants, populations) proportional to the amount of old stuff
- Rate rises faster and faster
- Quickly exceeds any reasonable limit
- Not very realistic!

# Logistic Growth

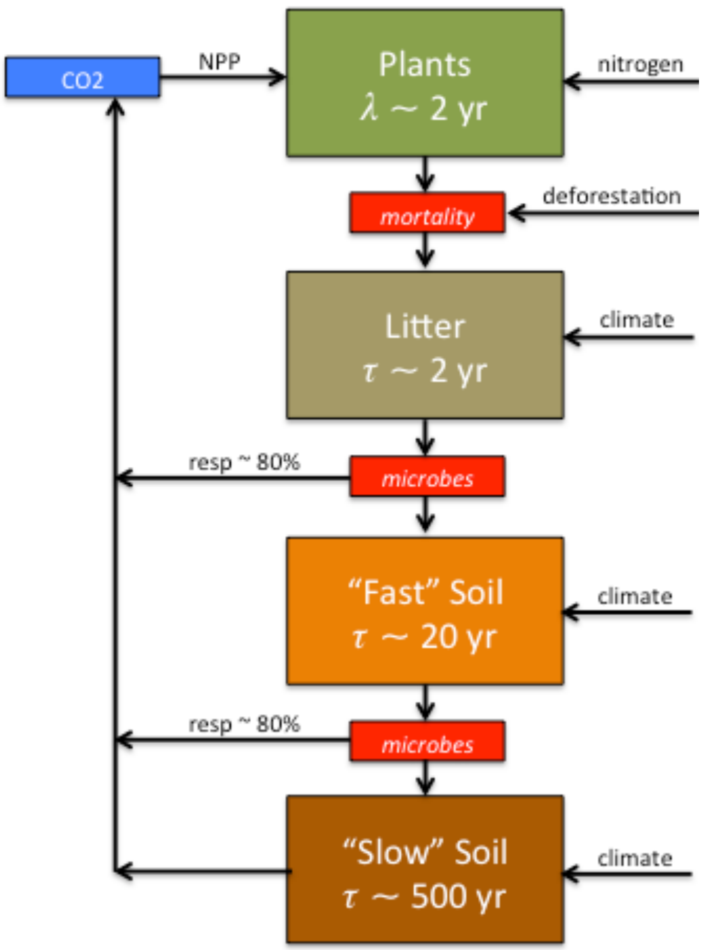
http://biocycle.atmos.colostate.edu/shiny/logistic/




- Let growth rate depend on population
- Exponential growth when population is small
- Growth rate shrinks as population approaches "carrying capacity"

# Cascading Carbon Pools

## Conceptual Model




## Mathematical Model

$$\frac{dP}{dt} = NPP - MORT$$

$$\frac{dL}{dt} = MORT - \frac{L}{\tau_L} Q_{10}^{\frac{T-T_0}{10}}$$

$$\frac{dF}{dt} = \left[ \frac{L}{\tau_L}(1-\epsilon) - \frac{F}{\tau_F} \right] Q_{10}^{\frac{T-T_0}{10}}$$

$$\frac{dS}{dt} = \left[ \frac{F}{\tau_F}(1-\epsilon) - \frac{S}{\tau_S} \right] Q_{10}^{\frac{T-T_0}{10}}$$

Symbols used:

- NPP = Net primary production (photosynthesis minus plant respiration, in GtC/yr)
- MORT = plant mortality (transfers carbon from live plants to dead litter, GtC/yr)
- $\tau_L$, $\tau_F$, $\tau_S$ = turnover time for carbon in litter, fast, and slow soil pools (years)
- $\epsilon$ = efficiency of microbial respiration (fraction of what they "eat"" that turns into CO2)
- $Q_{10}$ = fractional increase in decomposition rates per 10 degrees of warming
- T = temperature (Celsius), prescribed from a simple climate model using the IPCC SRES A2 emissions scenaro.

- Plants grow logistically limited by nutrients
- Carbon from dead plants becomes litter
- Litter decomposes quickly into fast and then slow soil organic matter

http://biocycle.atmos.colostate.edu/shiny/Land/

# Discretize the Model

## Replace all the *differentials* with *differences*

**Mathematical Model (differential equations)**

$$\frac{dP}{dt} = NPP - MORT$$

$$\frac{dL}{dt} = MORT - \frac{L}{\tau_L} Q_{10}^{\frac{T-T_0}{10}}$$

$$\frac{dF}{dt} = \left[\frac{L}{\tau_L}(1-\epsilon) - \frac{F}{\tau_F}\right] Q_{10}^{\frac{T-T_0}{10}}$$

$$\frac{dS}{dt} = \left[\frac{F}{\tau_F}(1-\epsilon) - \frac{S}{\tau_S}\right] Q_{10}^{\frac{T-T_0}{10}}$$

**Discrete Model (algebraic equations)**

$$\Delta P = (NPP - MORT)\Delta t$$

$$\Delta L = \left( MORT - \frac{L}{\tau_L} Q_{10}^{\frac{T-T_0}{10}} \right) \Delta t$$

$$\Delta F = \left[\frac{L}{\tau_L}(1-\varepsilon) - \frac{F}{\tau_F}\right] Q_{10}^{\frac{T-T_0}{10}} \Delta t$$

$$\Delta S = \left[\frac{F}{\tau_L} F(1-\varepsilon) - \frac{S}{\tau_S}\right] Q_{10}^{\frac{T-T_0}{10}} \Delta t$$

Approximate calculus with algebra

# Algorithm (Recipe)

Write down a procedure to update *(P,L,F,S)*

**Discrete Model (algebraic equations)**

$$\Delta P = (NPP - MORT)\Delta t$$

$$\Delta L = \left( MORT - \frac{L}{\tau_L} Q_{10}^{\frac{T-T_0}{10}} \right)\Delta t$$

$$\Delta F = \left[ \frac{L}{\tau_L}(1-\varepsilon) - \frac{F}{\tau_F} \right] Q_{10}^{\frac{T-T_0}{10}} \Delta t$$

$$\Delta S = \left[ \frac{F}{\tau_L} F(1-\varepsilon) - \frac{S}{\tau_S} \right] Q_{10}^{\frac{T-T_0}{10}} \Delta t$$

**Algorithm (arithmetic to update pools)**

$qmult = Q_{10}^{(T-T0)/10}\,\Delta t$

$P^{new} = P^{old} + (NPP - MORT)\,\Delta t$

$L^{new} = L^{old} + (MORT - L^{old}/\tau_L)\; qmult$

$F^{new} = F^{old} + ((1-\varepsilon)L^{old}/\tau_L - F^{old}/\tau_F)\; qmult$

$S^{new} = S^{old} + ((1-\varepsilon)F^{old}/\tau_F - S^{old}/\tau_S)\; qmult$

http://biocycle.atmos.colostate.edu/shiny/Land/

# Computer Program

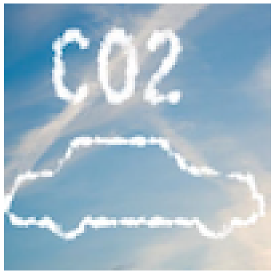
Convert arithmetic recipe into programming language

| Algorithm<br>(arithmetic to update pools) | Computer Program<br>(instructions in R) |
|---|---|
| $P^{new} = P^{old} + (NPP - MORT)\ \Delta t$ | `plant[i+1] <- plant[i] + NPP[i] - mortality[i]` |
| $L^{new} = L^{old} + (MORT - L^{old}/\tau_L)\ qmult$ | `litter[i+1] <- litter[i] + mortality[i] - litter[i]/tau.litter * resp.enhancement` |
| $F^{new} = F^{old} + ((1-\varepsilon)L^{old}/\tau_L - F^{old}/\tau_F)\ qmult$ | `fast.soil[i+1] <- fast.soil[i] + resp.enhancement * ( (1.-eff.microbes) * litter[i]/tau.litter - fast.soil[i]/tau.fast)` |
| $S^{new} = S^{old} + ((1-\varepsilon)F^{old}/\tau_F - S^{old}/\tau_S)\ qmult$ | `slow.soil[i+1] <- slow.soil[i] + resp.enhancement * ( (1.-eff.microbes) * fast.soil[i]/tau.fast - slow.soil[i]/tau.slow)` |

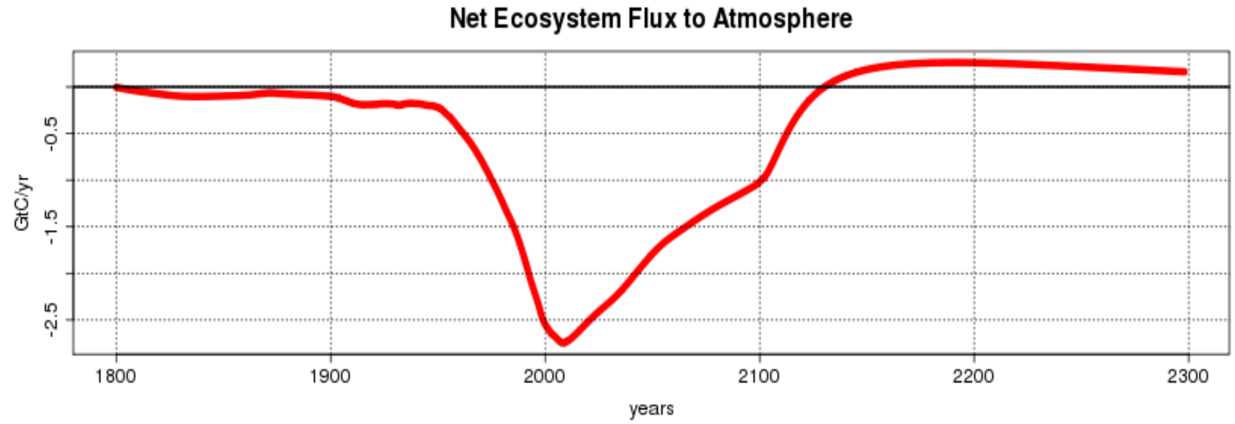
```
plant[i+1] <- plant[i] + NPP[i] - mortality[i]

litter[i+1] <- litter[i] + mortality[i]
               - litter[i]/tau.litter * resp.enhancement

fast.soil[i+1] <- fast.soil[i] + resp.enhancement * (
                  (1.-eff.microbes) * litter[i]/tau.litter -
                  fast.soil[i]/tau.fast)

slow.soil[i+1] <- slow.soil[i] + resp.enhancement * (
                  (1.-eff.microbes) * fast.soil[i]/tau.fast -
                  slow.soil[i]/tau.slow)
```

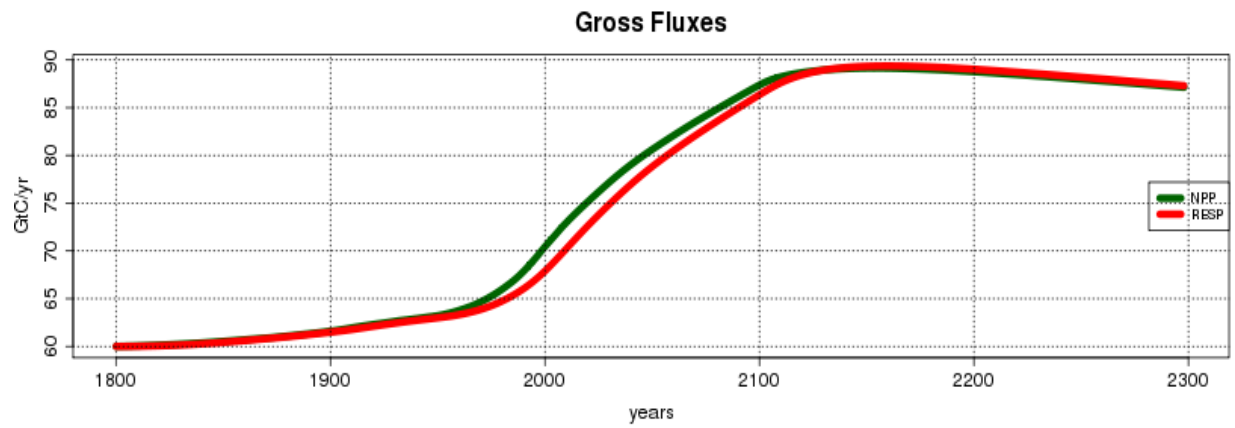
http://biocycle.atmos.colostate.edu/shiny/Land/

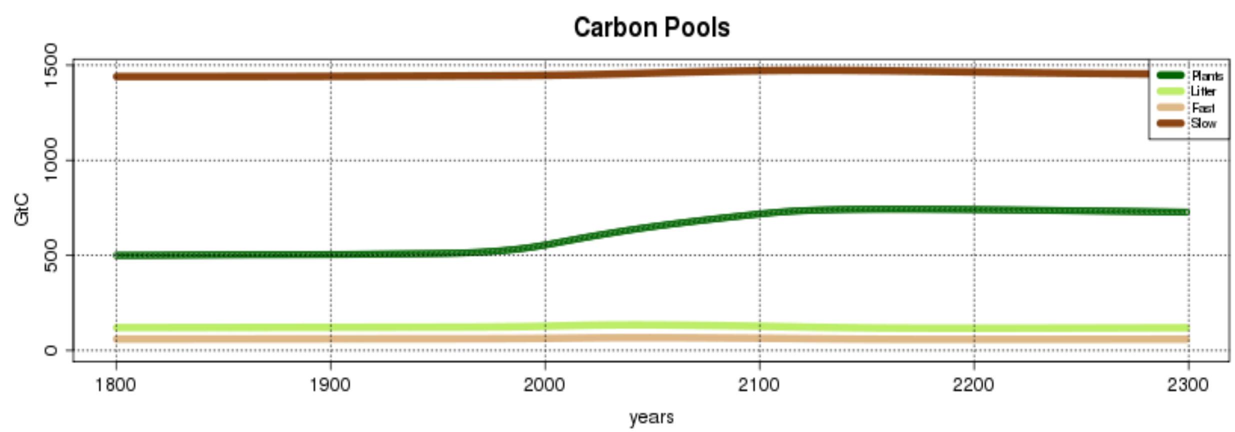
# Results








Try it yourself and test the effects of $CO_2$ fertilization, nitrogen deposition, deforestation, and climate!

http://biocycle.atmos.colostate.edu/shiny/Land/