

ATS 761: Land-Atmosphere Interactions

Assignment #1

Due Tues Sept 25

NOTE: these two problems are not hard, but may take you quite a lot of time if you haven't written computer programs in a long time. It will help focus your reading and class time if you read through the assignment and start thinking about how to solve these problems. I've outlined pretty much all you have to do below.

1. Write a simple computer program to calculate the diurnal cycle of incoming solar radiation (insolation) under cloudless skies for the following locations and dates:

- Fort Collins, CO USA on August 21 ($P = 855$ mb)
- Fort Collins, CO USA on February 21 ($P = 845$ mb)
- Mogadishu, Somalia on March 21 ($P = 1010$ mb)
- Wellington, New Zealand on October 21 ($P = 1010$ mb)

For each location and date above, make a graph of the surface insolation for each hour of the day from 0=midnight to 23=11 PM (local standard time).

Here's how:

In the absence of atmospheric scattering and absorption, the power of the Sun's radiation incident on a horizontal surface is

$$S_H = \frac{S_C}{r_v^2} \cos Z \quad (\text{Bonan eqn 4.4})$$

where $S_C = 1367 \text{ W m}^{-2}$ is the "solar constant," r_v is the "radius vector" (the ratio of the actual distance between the Sun and the Earth to its annual average), and $\cos Z$ is the cosine of the solar zenith angle. When the cosine of the solar zenith angle is negative, the Sun is below the horizon and we can just set the incoming solar radiation to zero.

The cosine of the solar zenith angle is

$$\cos Z = \sin \phi \sin \delta + \cos \phi \cos \delta \cos h \quad (\text{Bonan eqn 4.1})$$

where ϕ is latitude, δ is declination (latitude where the Sun is directly overhead at noon), and h is the *hour angle* ($h = 0$ at noon, $h = \pi/2$ at 6 PM, $h = \pi$ at midnight, $h = 3\pi/2$ at 6 AM).

To calculate the hour angle, simply compute

$$h = -\pi + \frac{LST}{24} 2\pi$$

where LST is local solar time.

Declination (latitude of the sub-Solar point) is given by

$$\delta = \varepsilon \sin \theta$$

where ε is the tilt of the Earth's axis (currently $\varepsilon = 0.4091$ radians) and θ is the angular (seasonal) position of Earth in its orbit. The seasonal position angle θ is measured from the vernal equinox (March 21), so that $\theta = \pi/2$ at the June solstice, $\theta = \pi$ at the autumnal equinox (Sept 21), and $\theta = 3\pi/2$ at the December solstice.

The radius vector depends on both the position of the Earth in its orbit and the "shape" of the orbit itself according to:

$$\frac{1}{r_v} = \left(\frac{R_0}{R_E} \right) = \frac{(1 - e^2)}{[1 + e \cos(\theta - \varpi)]}$$

where R_0 is the average Sun-Earth distance, R_E is the actual Sun-Earth distance, e is the eccentricity of Earth's orbit (deviation from a perfect circle), θ is the seasonal position angle (see above), and ϖ is the angle between the orbital position of the Earth's perihelion and the vernal equinox. Currently, $e = 0.0167$ and $\varpi = 1.7963$. See <http://en.wikipedia.org/wiki/Insolation> for a derivation and discussion.

The solar beam is also attenuated by atmospheric scattering and absorption. In a clear sky, incoming solar radiation at the Earth's surface can be approximated as:

$$S = S_H \tau^m \quad \text{(Bonan eqn 4.6)}$$

where S_H is the incident solar radiation on a horizontal surface above the atmosphere (defined above), τ is the atmospheric transmissivity, and m is the air mass through which the solar beam must pass. For this problem, assume $\tau = 0.7$. The air mass is

$$m = \frac{1}{\cos Z} \frac{P}{P_S} \quad \text{(Bonan eqn 4.7)}$$

where $\cos Z$ is the cosine of the solar zenith angle (defined above), P is atmospheric pressure and $P_S = 1013.25$ mb is the atmospheric pressure at sea level.

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2. Write a simple computer program to calculate the average “daily” variation of surface temperature on the Equator of the Moon!

Here’s how:

The Moon is “tidally locked” to the Earth so that the time required to rotate once on its axis is exactly the same as the period of its orbit around the Earth (that is, the length of the Moon’s “day” is the same as the length of a Lunar “month:” 29 Earth days, 12 hours, 44 minutes).

The Moon has no atmosphere, and the “tilt” of its axis relative to the orbital plane of the Earth-Moon system around the Sun is negligible. Therefore the calculation of insolation at the Lunar surface is much simpler than the one you did for the Earth in problem 1 above. Assuming that the tilt of the Moon’s axis is zero, $\delta = 0$. According to Bonan’s equation 4.1 (above) then, at the Moon’s equator the cosine of the solar zenith angle is just equal to the cosine of the hour angle. Much simpler than the Earth!

On Earth the surface energy budget is

$$\rho c \frac{\Delta T}{\Delta t} \Delta z = (S \downarrow - S \uparrow + L \downarrow - L \uparrow) - H - \lambda E = G \quad (\text{Bonan eqn 13.1})$$

On the left-hand side, ρ is the density of the soil (kg m^{-3}), c is the heat capacity of the soil ($\text{J m}^{-3} \text{K}^{-1}$), ΔT is the change in soil temperature (Kelvin) over a time step (Δt , seconds) for each soil layer whose thickness is Δz (meters). On the right-hand side the four terms inside the parentheses are shortwave radiation down and up, and longwave radiation down and up. H is the turbulent flux of sensible heat, l is the latent heat of evaporation of water, E is the rate of evaporation of water from the surface, and G is the change in ground heat storage. All terms in this equation have the units of W m^{-2} .

The Moon’s surface energy budget is much simpler! There’s no water to evaporate and no turbulent air to carry sensible heat. There’s a little bit of longwave radiation emitted by the Earth that heats the Moon’s surface, but on the far side (which we never see) this is zero so we can neglect this. All of the upward flux of shortwave radiation is just the incoming solar radiation times the Moon’s albedo. To calculate upward longwave emission we can assume the surface emits as a blackbody. So the Moon’s surface energy budget is

$$\rho c \frac{\Delta T}{\Delta t} \Delta z = S \downarrow (1 - \text{albedo}) - \sigma T_s^4$$

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In visible wavelengths, the Moon is surprisingly dark, with an albedo of just 7% (like asphalt!). Across the whole Solar spectrum it's a little brighter, so please use the broadband albedo of 11%.

To calculate the temperature, divide the Lunar "soil" into 15 layers. Make the top layer thinnest (0.02 m = 2 cm), and then make each layer underneath 1.5 times as thick as the one above. This will get you 15 lunar soil temperatures down to about 17.5 meters depth.

The downward flux of heat at the top of each layer is proportional to the difference in temperature between adjacent layers:

$$F \uparrow = -k \frac{\Delta T}{\Delta z} \quad (\text{Bonan eqn 9.1})$$

The proportionality constant k is called the "thermal conductivity." The negative sign is there because the layers are counted down from the surface but the heat flux is defined as positive upward, away from the surface.

Changes in soil temperature are determined by the difference in the heat flux into and out of each layer. This is called the "heat flux divergence:"

$$\rho c \frac{\Delta T}{\Delta t} \Delta z = -(F_z - F_{z+\Delta z}) \quad (\text{Bonan eqn 9.2})$$

There's a nice worked example of this on Bonan page 133. The book also gives typical values of heat capacity and thermal conductivity for different kinds of soil.

The Lunar "soil" is actually made up of mineral grains blasted to smithereens by billions of years of impact bombardment. Unlike Earth soils, the pore space is not filled with water or air. It's just vacuum, so the heat flow through the lunar soil is much slower than on Earth, depending on conductance and thermal radiation between adjacent grains!

According to Colozza (1991), the volumetric heat capacity of lunar regolith is about 4.4 million $\text{J m}^{-3} \text{K}^{-1}$. Use this value for ρc in the equation above. The thermal conductivity of lunar "soil" is only about $0.025 \text{ W m}^{-1} \text{K}^{-1}$. This is about 100 times less conductive than soils on Earth!

To calculate the changes in temperature with depth and time you'll need to add solar radiation to the surface, subtract upward longwave, and step the heat flux equations forward in time. You'll find that if you use a time step that's too long, the numerical solution will become unstable and you'll get crazy values (for example negative temperatures Kelvin). I find I can get away with a time step of about an hour.