Surface Energy Budget

Please read Bonan Chapter 13

Energy Budget Concept

- For any system,
  \[(\text{Energy in}) - (\text{Energy out}) = (\text{Change in energy})\]

- For the land surface,
  - Energy in = ?
  - Energy Out = ?
  - Change in energy = ?

Energy Budget Concept

- For the land surface,
  - Energy in = Radiation
  - Energy Out = Radiation + Turbulent fluxes of “sensible” and “latent” heat
  - Change in energy = changes in temperature of soil, plants, water, and air

Surface Radiation Budget

- Shortwave
  - Down (solar constant, seasonal and diurnal geometry, atmospheric attenuation, clouds and aerosol)
  - Up (albedo)

- Longwave
  - Down (emission from atmosphere depends on temperature profile, water vapor, clouds)
  - Up (surface temperature, emissivity)
Surface Radiation Budget

\[ R_{\text{net}} = SW \downarrow - SW \uparrow + LW \downarrow - LW \uparrow \]
\[ = SW \downarrow (1 - \text{albedo}) + LW \downarrow (1 - \varepsilon) + \varepsilon \sigma T_a^4 \]

- **Shortwave**
  - Down (solar constant \( \times \cos Z \times \text{transmissivity} \times \text{airmass} \))
  - Up (albedo \( \times \) SW down)

- **Longwave**
  - Down (complicated! Weighted average of \( \sigma T_a^4 \))
  - Up (\( \varepsilon \sigma T_s^4 \))

Radiation Budget Components

- Net solar follows \( \cos(z) \)
- LW fluxes much less variable \( \varepsilon \sigma T_a^4 \)
  - LW up follows surface T as it warms through day
  - LW down changes little
  - LW net opposes \( SW_{\text{net}} \)
- \( R_{\text{net}} \) positive during day, slightly negative at night

Land Surface Energy Budget

- Very little of the energy gained by net radiation is stored in the ground \( (G) \)
- Most is emitted as LW IR and turbulent fluxes of sensible \( (H) \) and latent heat \( (LE) \)
- Latent energy is then released into atmosphere when vapor condenses

Surface Energy Budget

Storage change = Energy in – energy out

\[ \rho c \frac{\Delta T}{\Delta t} \Delta z = (S \downarrow - S \uparrow + L \downarrow - L \uparrow) - H + \lambda E = G \]

\[ R_{\text{net}} = (S \downarrow - S \uparrow) + (L \downarrow - L \uparrow) = H + \lambda E + G \]

Role of the land surface:
Partition of net radiation into turbulent fluxes & storage
Surface Energy Budgets

- \( R_{\text{net}} = H + LE + G \)  
- Daytime turbulent fluxes upward  
- Night: turbulent fluxes downward (dew or frost!)
- Dry surfaces \( R_{\text{net}} \sim H \)  
- Wet surfaces \( R_{\text{net}} \sim LE \)

Heat Fluxes ~ Currents

- Sensible heat flux  
  - Driving potential is a difference in temperature  
  - \( H \) is proportional to \( \Delta T \)
- Latent heat flux  
  - Driving potential is a difference in vapor pressure  
  - \( LE \) is proportional to \( \Delta e \)

Remember Ohm’s Law?

- Flow of current through a resistor is ratio of difference in potential to resistance (this is just a “definition of resistance”)  
- This is another form of our familiar concept of stuff flowing from high concentration to low concentration (like “Fickian Diffusion”)

Sensible Heat Flux

- Driving potential is a difference in temperature  
- \( H \) is proportional to \( \Delta T \)

\[
H = \rho c_p \rho \frac{\Delta T}{r} = \rho c_p \frac{T_S - T_a}{r}
\]

\[
\text{Watts m}^{-2} = \frac{(\text{kg m}^{-3})(\text{J K}^{-1} \text{kg}^{-1})(\text{K})}{(\text{s m}^{-1})} = \frac{(\text{kg} m^{-3})(\text{J K}^{-1} \text{kg}^{-1})(\text{K})}{(\text{s m}^{-1})} = \frac{\text{J s}^{-1}}{\text{m}^2} = \text{W m}^{-2}
\]
**Water Vapor Pressure**

- Molecules in an air parcel all contribute to pressure.
- Each subset of molecules (e.g., N$_2$, O$_2$, H$_2$O) exerts a *partial pressure*.
- The VAPOR PRESSURE ($e$), is the pressure exerted by water vapor molecules in the air.

**Latent Heat Flux**

- Driving potential is a difference in water vapor pressure.
- LE is proportional to $\Delta e$.

$\text{LE} = \rho c_p \frac{\Delta e}{r} = \rho c_p \frac{e_s - e_a}{\gamma r}$

- $\Delta e$: difference in vapor pressure (Watts m$^{-2}$)
- $\gamma$: *Psychrometric constant*: $\gamma = (C_p P)/(0.622 \bar{\alpha})$
- $\bar{\alpha}$: psychrometric constant, 66.5 Pa °C$^{-1}$
- $r$: aerodynamic resistance, s m$^{-1}$

**Molecular Structure of Water**

Water's unique molecular structure and hydrogen bonds enable all 3 phases to exist in Earth's environments.

**“Latent” (hidden) Energy associated with phase changes**

Water vapor changes phase as follows:
- **Solid**: Add 80 calories
- **Liquid**: Add 40 calories
- **Gas**: Remove 640 calories
- **Freezing**: Remove 80 calories
- **Melting**: Add 80 calories
- **Cooling**: Add 100 calories
- **Condensing**: Remove 640 calories
- **Evaporating**: Add 40 calories

Latent heat of fusion = 80 calories
Latent heat of vaporization = 640 calories
Why does it take so much energy to evaporate water?

- In the liquid state, adjacent water molecules attract one another.
- This same hydrogen bond accounts for surface tension on a free water surface.

"plus" charge on hydrogen in one water molecule attracts the "minus" charge on a neighbor’s oxygen.

Evaporation must break these hydrogen bonds.

Clausius-Clapeyron Eqn (see Monteith & Unsworth, pp 11-13)

- From Second Law of Thermodynamics:

\[ \frac{d e_s}{dT} = \frac{L}{T} \left( \frac{\alpha_2 - \alpha_1}{\alpha_2} \right) \]

\( d e_s \) = change in saturation vapor pressure
\( \frac{d e_s}{dT} \) = Latent heat of vaporization
\( \frac{L}{T} \left( \frac{\alpha_2 - \alpha_1}{\alpha_2} \right) \) = specific volumes of liq & vapor

- Approximate but very accurate 0\(^\circ\) to 35\(^\circ\) C

\[ e_s(T) = e_s(T^*) \exp \left\{ A(T - T^*)/(T - T^*) \right\} \]

where \( A = 17.27 \), \( T^* = 273 \, \text{K} \) \( (e_s(T^*) = 0.611 \, \text{kPa}) \), and \( T^* = 36 \, \text{K} \).

Water Vapor Saturation

- Water molecules move between the liquid and gas phases.
- When the rate of water molecules entering the liquid equals the rate leaving the liquid, we have equilibrium.
- The air is said to be saturated with water vapor at this point.

Saturation and Temperature

- The saturation vapor pressure of water increases with temperature.
  - At higher T, faster water molecules in liquid escape more frequently causing equilibrium water vapor concentration to rise.
  - We sometimes say "warmer air can hold more water."
- There is also a vapor pressure of water over an ice surface.
  - The saturation vapor pressure above solid ice is less than above liquid water.
**Latent Heat Flux**
- Driven by difference in vapor pressure
- Over open water $e_{\text{surface}} = e_{\text{sat}}(T_s)$
- Over vegetation, liquid water is evaporating inside tiny openings in leaves called *stomata* (singular “*stomate*”)
- Evapotranspiration = latent heat flux is driven by the vapor pressure deficit
  \[ \text{vpd} = (e_{\text{sat}}(T_s) - e_a) \]
  \[ \text{LE} = \frac{\rho c_p}{\gamma} \frac{e_s - e_a}{r} = \frac{\rho c_p}{\gamma} \frac{e_{\text{sat}}(T_s) - e_a}{r} \]

**Idealized Diurnal Cycle**
- $R_{\text{net}}$ follows $\cos(z)$ during day, negative at night (LW cooling)
- Downward turbulent fluxes at night
- Ground heat flux smaller: downward during day and up at night
  \[ R_{\text{net}} = H + \text{LE} + G \sim H + \text{LE} \]

**Energy Budget Components**

**Global Variations**
- Tropics -> pole gradients
- Lower albedo over oceans (higher $R_{\text{net}}$)
- $G \sim 0$: Storage negligible over land
- Water budget in oceans and required *atmospheric transport*
Radiation, Hydrology, & the Sfc Energy Budget

- Wet places: evaporation balances radiation
- Dry places: evaporation balances precipitation
- Smooth transition in between

Seasonal Energy Budgets

- Seasonal course of $R_{net}$ due to Sun-Earth geometry
- Moist climates feature near balance of $R_{net} \sim LE$
- Dry climates feature near balance of $R_{net} \sim H$
- Others are intermediate
  - Spring vs fall in Texas
  - Summer (leaves) vs spring and fall in Wisc
- $(H, LE) >> G$ everywhere

Annual Radiation and Hydrology

- Radiation favors evaporation over runoff
- Dry (sunny) places: LE ~ Precipitation
- Wet (cloudy) places: Runoff ~ Precipitation

Seasonal Energy Fluxes
**Diurnal Variations**

Tropical Forest: Rondonia, Brazil (10° S)

- Precipitation
- Solar radiation
- Albedo
- Net radiation
- Latent heat flux
- Sensible heat flux

**Partition of Net Radiation**

- Energy budget “closure”
- Forest “harvests” radiation to extract water from soil
- Grassland passes more energy back to atmosphere as sensible heat

**Diurnal Energy and CO₂**

- CO₂ flux is a mirror image of LE (stomatal control)
- Physiological differences: broadleaf vs needleleaf

**Partition of Net Radiation**

\[ R_{\text{net}} = (S \downarrow - S \uparrow) + (L \downarrow - L \uparrow) = H + \lambda E + G \]

- Sensible flux driven by ΔT
- Latent flux driven by VPD
- Ground heat flux

\[ \gamma = (C_p \rho)(0.622) = 66.5 \text{ Pa} \text{ °C}^{-1} \]

“Psychrometric constant”
Surface Energy Budget

Energy in = energy out + storage change

\[ (1-r)S_1 + eL_1 = \varepsilon \sigma (T_s + 273.15)^4 + H + \Delta E + G \]

\[ (1-r)S_1 + eL_1 = \varepsilon \sigma (T_s + 273.15)^4 - \rho C_P \frac{(T_a - T_s)}{r_{H}} + \frac{\rho C_P (e_s - e_a)}{r_w} + h_c \frac{(T_s - T_a)}{\Delta z} \]

- Can “solve” for surface temperature
- Physical properties: albedo, emissivity, heat capacity, soil conductivity & temperature
- “Resistances” are properties of the turbulence … depend sensitively on \( H \! \)!

Penman-Monteith Equation

“Thermodynamic” energy balance

\[ \lambda E = (R_n - G) - H = (R_n - G) + \rho C_P (T_a - T_s) / r_H \]

“Turbulent” energy balance

\[ \lambda E = \rho C_p (e_s - e_a) / r_w \]

\[ \varepsilon_s(T) = \varepsilon_a(T) + \delta (T_s - T_a) \]

VPD approximated by linearization of Clausius-Clapeyron equation

\[ T_s - T_a = (r_H / \rho C_p) (R_n - G - \lambda E) \]

Clausius-Clapeyron Eqn

(see Monteith & Unsworth, pp 11-13)

- Approximate but very accurate 0°C to 35°C

\[ e_s(T) = e_a(T^*) \exp \left[ A(T - T^*) / (T - T^*) \right] \]

where \( A = 17.27, T^* = 273 \) K \( (e_a(T^*) = 0.611 \) kPa), and \( T^* = 36 \) K.

- **Slope** \( s = \Delta e_s / \Delta T \) accurate from 0°C to 40°C

\[ s = \lambda M_w e_s(T) / (RT^2) \]

\[ \lambda = 2.48 \) J kg\(^{-1} \), \( M_w \) (mol wt water) = 0.018 mol kg\(^{-1} \)

\[ R = 8.314 \) J mol\(^{-1} \) K\(^{-1} \) (“universal gas constant”)

Solutions to P-M Equation

Latent heat flux

\[ \lambda E = \delta (R_n - G) + \rho C_P (e_s - e_a) / (r_H / r_w) \]

Sensible heat flux

\[ H = (R_n - G) \gamma^* - \rho C_P (e_s - e_a) / (r_H / r_w) \]

Surface temperature

\[ T_s = T_a + \frac{(R_n - G) \gamma^* / \rho C_P - (e_s - e_a)}{s + \gamma^*} \]
Turbulent Fluxes

Please read Bonan, Chapter 14
Richardson’s Rhyme

- “Big whorls have little whorls,
  Which feed on their velocity;
  And little whorls have lesser whorls,
  And so on to viscosity”
  - Lewis Richardson, The supply of energy from and to Atmospheric Eddies 1920

- “Great fleas have little fleas
  Upon their backs to bite ‘em,
  And little fleas have lesser fleas,
  And so, ad infinitum”
  - Augustus De Morgan
    (19th century mathematician, parodying Jonathon Swift, 1733)

Sonic Anemometer

- Measures elapsed time for sound pulses to cross air in 3D
- Speed of sound is a known function of temperature
- Relative motion determined accurately in 3D
- Very fast instrument response time

Time Series of Turbulence

\[
\begin{align*}
\bar{a} &= \bar{a} + \bar{d} \\
\sigma^2 &= \frac{1}{N} \sum (a_i - \bar{a})^2 = \bar{d} \bar{d}' \\
\bar{a} \bar{b} &= \bar{a} \bar{b} + \bar{a} \bar{b}' \\
\text{cov}(a, b) &= \frac{1}{N} \sum (a_i - \bar{a})(b_i - \bar{b}) = \bar{a} \bar{b}'
\end{align*}
\]

Turbulent Heat Flux

\[w' < 0 \quad T' < 0\]
\[w' > 0 \quad T' > 0\]

- Imagine a turbulent eddy over a hot surface
- Updrafts are systematically warmer than downdrafts

- Updraft: \[\frac{w'}{T'} > 0\]
- Downdraft: \[\frac{w'}{T'} > 0\]
**Sensible Heat Flux (Reynolds’ Averaging)**

Upward sensible heat flux $H = c_p \rho w T$

- Mean of a mean is the mean
- Mean of a prime is zero
- Mean of a product is not necessarily zero

Temp (K)

Heat capacity at constant pressure

Density of air $p/RT$ (~ 1.2 kg m$^{-3}$)

Vertical velocity (m s$^{-1}$)

$w = \bar{w} + w'$, $T = \bar{T} + T''$

$\bar{w}T = (\bar{w} + w')(\bar{T} + T'')$

$\bar{w}T = \bar{w} \bar{T} + \bar{w} T'' + w' \bar{T} + w' T''$

$\bar{w}T = \bar{w} \bar{T} + \bar{w} T'' + w' \bar{T} + \bar{w} T''$

$\bar{w}T = \bar{w} \bar{T} + \bar{w} T'' + \bar{w} \bar{T} + \bar{w} T''$

**Turbulent Fluxes**

**Sensible Heat Flux**

$\bar{w}T = \bar{w} \bar{T} + \bar{w} T''$

Total = mean + eddy

$\bar{w}q = \bar{w} \bar{q} + \bar{w} q'$

Latent Heat Flux

$w \varphi = \bar{w} q + w q'$

- Near the ground, $\bar{w} = 0$
- “Eddy” terms dominate
- How can these fluxes be measured?

Weird units:

- K m s$^{-1}$
- kg kg$^{-1}$ m s$^{-1}$

**Variance, Covariance, Correlation**

$\overline{c} = \frac{1}{N} \sum \bar{c}_i$ mean

$\bar{c}_i = c_i - \overline{c}$ perturbation

$\bar{c} = \frac{1}{N} \sum \overline{(c_i - \overline{c})(c_j - \overline{c})}$ variance

$w'^2 = \sigma_w^2$ variance of $w$

$\bar{w}'c' = \frac{1}{N} \sum \overline{(w_i - \overline{w})(c_j - \overline{c})}$ covariance of $w$ and $c$

$\frac{\bar{w}'c'}{\sigma_w \sigma_c} = r$ Normalized covariance is the correlation coefficient

**FluxNet: An international “network of networks” > 550 sites**

- 10 Hz measurements from many sites for > 5 yrs
- $H$, LE, and NEE of CO$_2$ at most sites
- Data available online: [http://fluxnet.ornl.gov](http://fluxnet.ornl.gov)
Surface Layer Mixing

- Turbulent eddies near the surface act to mix atmospheric properties ($T$, $q$, $u$) and reduce vertical gradients.
- Assume a characteristic length scale $l'$ for eddy mixing, then
  \[ u = u(z) + u' \]
  \[ u' = -l' \frac{\partial \bar{u}}{\partial z} \]
  If eddies are isotropic (length and depth similar), then
  \[ w' \sim u' \]
  so
  \[ w' \sim l' \frac{\partial \bar{u}}{\partial z} \]

Surface Layer Stress

\[ \tau_z = -\rho \bar{u} u' \]
\[ = -\rho \left( -l' \frac{\partial \bar{u}}{\partial z} \right) \left( -l' \frac{\partial \bar{u}}{\partial z} \right) = -\rho l'^2 \frac{\partial \bar{u}}{\partial z} \]
Define $u_* = \sqrt{\frac{\tau_z}{\rho l'^2}}$ then
\[ \frac{\tau_z}{\rho} = \frac{K_u}{\rho} \frac{\partial \bar{u}}{\partial z} = u_*^2 \]

Surface Layer (cont’ d)

- Near the surface, eddies are limited in size by the proximity of the ground, so $l'$ in $K_u$ is $l'(z)$
- Assume $l' = k z$, where $k \approx 0.4$ is an empirical coefficient known as “von Karman’s constant”
- Leads to a characteristic relationship for variation of mean wind speed with height:
  the log-wind profile

Log Wind Profile

- Mean wind speed in the surface layer is decelerated by friction whose influence is felt aloft through eddy momentum flux
- Varies logarithmically with height
- $Y$-intercept of log-linear plot of SL wind vs $z$ is $z_0$, which we define as the “roughness length”

\[ \frac{\partial \bar{u}}{\partial z} = \frac{u_*}{k z} \]
\[ \int \frac{\partial \bar{u}}{\partial z} = \frac{u_*}{k} \ln \frac{z}{z_0} \]
\[ \bar{u}(z) = \frac{u_*}{k} \ln \frac{z}{z_0} \]
y-intercept of log-linear plot of SL wind vs $z$ is $z_0$, which we define as the “roughness length”

<table>
<thead>
<tr>
<th>Surface</th>
<th>Roughness length (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soil</td>
<td>0.001–0.01</td>
</tr>
<tr>
<td>Grass</td>
<td>0.003–0.01</td>
</tr>
<tr>
<td>Short</td>
<td>0.04–0.10</td>
</tr>
<tr>
<td>Tall</td>
<td>0.04–0.20</td>
</tr>
<tr>
<td>Crop</td>
<td>1.0–6.0</td>
</tr>
<tr>
<td>Forest</td>
<td>0.4–1.2</td>
</tr>
<tr>
<td>Low density</td>
<td>0.8–1.8</td>
</tr>
<tr>
<td>High density</td>
<td></td>
</tr>
<tr>
<td>Urban</td>
<td></td>
</tr>
<tr>
<td>Short building</td>
<td>1.5–2.5</td>
</tr>
<tr>
<td>Tall building</td>
<td>2.5–10</td>
</tr>
</tbody>
</table>

For neutral conditions
$$(z/z_0)\partial u/\partial z = 1/k$$

The Monin-Obukhov similarity theory states that when scaled appropriately the dimensionless mean vertical gradients of wind (u), temperature (T), and specific humidity (q) are unique functions of a buoyancy parameter ($\zeta$):

$$\phi_u = \phi_u(\zeta) = (1 - 16\zeta)^{-1/2}$$

for $\zeta < 0$ (unstable)

$$\phi_T = \phi_T(\zeta) = 1 + 5\zeta$$

for $\zeta \geq 0$ (stable)

- Empirical adjustment of log-wind profiles to account for buoyancy fluxes (anisotropy)

Estimation of Turbulent Fluxes

- Fluxes are driven by gradients in $u$, $T$, and $q$
- Fluxes are proportional to friction velocity
- These are simply definitions of $K_M$, $K_H$, $K_W$
- Ohm’s Law combined with Similarity
The Atmospheric Boundary Layer
(a.k.a ABL, PBL, CBL, NBL, SBL ...)

Free Atmosphere
Troposphere
Capping Inversion
Boundary Layer
Earth

Height, z
Horizontal distance, x

"The layer of atmosphere in turbulent contact with the surface"

PBL Temperatures
Diurnal Cycle
- Morning inversion broken by surface heating
- Shallow ML by 10 AM under RL from yesterday
- Superadiabatic surface layer at 2 PM
- New inversion forms near surface by 6 PM
- Nocturnal BL grows "from the bottom up"

PBL Wind Speeds
Annual Mean Diurnal Cycles
- Surface winds are maximum at midday
- Winds aloft are maximum at night (decoupling)
- Momentum mixing during daytime allows surface friction to be "felt" throughout ML

Typical Diurnal Cycle of PBL Over Land
Free Atmosphere
Capping Inversion
Mixed Layer
Residual Layer
Stable BL

Day 1 Night 1 Day 2
(Stull, 1988)
**CBL Development**

- Phase 1: Nocturnal Inversion Burn-off
- Phase 2: Rapid Rise
- Phase 3: Quasi-steady

- LCL
- Z:
- sunrise
- mid-morning
- mid-afternoon

**Boundary Layer Clouds**

- Big thermals that reach lifting condensation level are often capped by shallow cumulus clouds.
- If these clouds are forced to the level of free convection, they grow on their own by condensation heating.
- PBL-top clouds are an important means for venting PBL air into the free troposphere.