Land Surface Hydrology Parameterization for Atmospheric General Circulation Models Including Subgrid Scale Spatial Variability

D. ENTEKHABI AND P. S. EAGLESON

Department of Civil Engineering, Massachusetts Institute of Technology, Cambridge, Massachusetts

(Manuscript received 6 September 1988, in final form 1 February 1989)

ABSTRACT

Parameterizations are developed for the representation of subgrid hydrologic processes in atmospheric general circulation models. Reasonable a priori probability density functions of the spatial variability of soil moisture and of precipitation are introduced. These are used in conjunction with the deterministic equations describing basic soil moisture physics to derive expressions for the hydrologic processes that include subgrid scale variation in parameters. The major model sensitivities to soil type and to climatic forcing are explored.

1. Introduction

General circulation models (GCMs) promise to be valuable research tools in approaching large-scale hydroclimatological problems. However, many of the sensitive components of these numerical models are unresolved at the scales of the grid discretization that are computationally feasible in the near term. Such subgrid processes must, therefore, be parameterized within the model.

The GCMs used for climate research have time and space resolution coarser than those used in numerical weather prediction. Furthermore, climate models are integrated over much longer periods. The land surface–atmosphere interaction and the hydrological cycle are thus especially important to GCMs used in climate research. The parameterizations developed here are, therefore, especially targeted for climate simulation GCMs.

Of the parameterized components, the fluxes at the surface are of particular interest to hydrologists. Most GCMs in current use are equipped with rather simplistic, one-dimensional, empirical runoff ratio and evapotranspiration efficiency functions (Carson 1982). (Runoff ratio is the ratio of the surface precipitation excess to the incident precipitation; evapotranspiration efficiency is the ratio of the actual to the potential evapotranspiration.) Given that today's GCMs have grid areas larger than typical storm or basin areas, important subgrid scale variabilities at the ground lead to misrepresentation of the fluxes by the simple, one-dimensional formulas.

Corresponding author address: Dara Entekhabi, Ralph M. Parsons Laboratory, Department of Civil Engineering, Massachusetts Institute of Technology, Cambridge, MA 02139.

© 1989 American Meteorological Society

In this paper we derive runoff ratio and evapotranspiration efficiency functions that incorporate spatial variability through simple assumptions about the statistical distribution of the relevant parameters at the subgrid scale. Throughout we strive for computational economy, use only dimensionless variables, and remain as true as possible to the physics of the problem.

Vertical soil infiltration and exfiltration processes are affected by gravity and soil capillary forces. The parameterizations introduced here incorporate both these effects and demonstrate the relative magnitude of each for different soil types and wetness conditions. The storage and redistribution of moisture in the layered soil column are processes that may be modeled independently of the parameterizations developed here (Abramopoulos et al. 1988).

2. Infiltration and runoff

a. *Mechanics of runoff generation*

Surface runoff is generated, for the most part, by the independent interaction of several spatially variable processes. The excess of precipitation intensity over soil infiltrability at a point, and the occurrence of precipitation over saturated and impermeable surfaces have been identified as two major mechanisms of inducing surface runoff. The former type of runoff is generally referred to as Horton runoff in the hydrologic literature (Freeze 1974). Its occurrence is limited to localized areas of low permeability experiencing intense rainfall; otherwise and commonly, the infiltrability of soils exceeds precipitation. Hence only a small fraction of the surface runoff contribution to streamflow may be accounted for by Horton runoff. The latter type of surface runoff mentioned here is largely responsible for the rapid response of streams to precipitation.

Hewlett (1961) and Hewlett and Hibbert (1965) suggested that expanding saturated zones along hill and valley bottoms result in a *partial area* contribution to storm runoff. All precipitation over these areas that are saturated by the rising shallow water table becomes surface runoff. Dunne and Black (1970) later produced observational evidence of this partial area contribution, and in most hydrologic literature the term Dunne runoff is employed to denote this mechanism of surface flow generation. We will incorporate both the Horton and Dunne processes in our parameterization of surface runoff.

For GCM grid area scales, the area-average precipitation intensity rarely assumes magnitudes exceeding the area-averaged soil infiltrabilities. The conventional spatial averaging inherent in GCM area discretization thus restricts the generation of surface runoff to those rare cases of intense precipitation over grid areas of low average permeability and high average water table. Subdivision of the GCM grid into smaller units of variable properties has been explored by Koster et al. (1988) as a solution to this problem. They find the technique useful for off-line sensitivity studies but costly for use in operational GCMs.

An alternative approach to the increasing of GCM resolution or the further subdivision of the landsurface component into smaller units is to regard a few relevant grid prognostics as the area-means of variables that have subgrid variance in the manner of Warrilow et al. (1986). This is analogous to the treatment of temporal variability by Eagleson (1978a, c) in his one-dimensional model of the average annual water balance. The current focus on surface runoff generation warrants treating the grid precipitation and the surface soil layer's hydraulic state as such distributed parameters.

b. Fractional wetting by precipitation

Precipitation over the grid area is determined by the combined, and sometimes coupled, moist-convective and large-scale condensation schemes of GCMs. Generally, fractions of grid volumes are treated as parcels that are forced to rise until convective stabilization governs the entire affected atmospheric profile. The fraction may be a constant or a variable, in which case it is dependent on convergence and vertical fluxes in the lower troposphere. Vapor condensates are allowed to evaporate as they fall through the atmospheric layers. Separately, and often simultaneously, any cases of supersaturation within the grid volumes are regarded as large-scale condensation. After satisfying all the conditions of precipitation formation and reevaporation, any residual at the lowest atmospheric layer is regarded as precipitation reaching the landsurface and is distributed uniformly over the entire grid area. Given the typically large area of individual GCM surface grids and the long time increments in GCM integration, the model precipitation intensities reaching the surface are generally unrealistically low. Furthermore, given the scale of typical GCM grid areas, storm coverages may only be fractional. We thus recognize the need for fractional wetting parameterization within GCMs.

Let us define that, at any time for each grid, a fraction κ of the grid area is affected by precipitation reaching the surface. The parameter κ may be related to the plume fraction of the grid volume that is active during moist-convective events. Theoretical and observational studies of κ are found in the literature (Eagleson 1984; Eagleson and Wang 1985; Eagleson et al. 1987). We follow Warrilow et al. (1986) in assuming that over fraction κ of the GCM grid area, the point precipitation intensity, P, is exponentially distributed, with mean $E(P)/\kappa$, as in

$$f_P(P) = \frac{\kappa}{E(P)} e^{-\kappa P/E(P)}, \quad P > 0. \qquad (1)$$

The parameter κ represents the scaling necessary to redistribute the GCM grid precipitation over the typical scale of precipitation events (mesoscale). Depending on the governing GCM spatial and temporal resolution, κ ranges between zero and one. The expectation of precipitation over the entire grid area, $E(P)$, is taken as the GCM resultant simulation of all precipitation reaching the landsurface, due to moist-convective and large-scale condensation events, at any time step over any grid area. Figure 1 illustrates the observed spatial variability of total storm depth for air mass thunderstorm rainfall in Arizona (Eagleson et al. 1987) in which case κ is about 0.66. Over the storm-affected κ fraction of the grid area, the point precipitation intensity is distributed such that there are lesser areas of high intensity corresponding to the tail of the probability density function (pdf) in (1).

c. Spatial heterogeneity of soil hydraulic conditions

The ground hydrology in current GCMs is modeled using a layered system of soils with known hydraulic

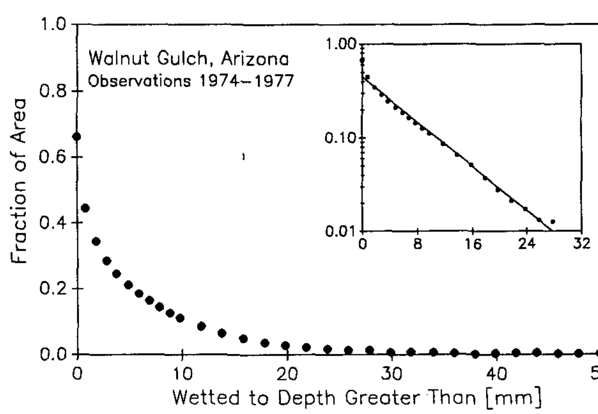

FIG. 1. Spatial distribution of total storm depth of airmass thunderstorm rainfall over a basin in Arizona. (Adapted from Eagleson et al. 1987.) Inset figure illustrates a log-linear curve fit to the data.

properties. For each grid area the soil of each layer is usually defined as a hybrid yet spatially invariant type and is characterized by a spatially uniform pattern of water content.

Important hydrologic processes depend on the heterogeneity of soil hydraulic conditions. At the soil surface, infiltration and exfiltration, hence runoff and bare soil evaporation, depend on the distributed nature of soil properties and states. These include type, texture, permeability, slope, elevation, and water content.

The surface runoff process is strongly and nonlinearly dependent on the soil water content in particular. The Horton component of surface runoff is the residual of precipitation intensity over the soil infiltrability, and the latter is highly sensitive to soil moisture conditions. The Dunne component of surface runoff is directly related to localized areas where the moisture content θ in the first soil layer equals the saturation value θ_{sat}. This chiefly applies to the low-land areas along stream channels, depressions, focal regions of closed drainage basins, zones of overall low soil thickness, high water table, and other impermeable surfaces. Spatial variability of surface soil moisture content also results from lateral moisture redistribution in a sloped drainage network. Areas of lower altitude and close to seepage faces tend to accumulate moisture. In addition, low permeability zones such as those associated with rock outcrops, crusted soils, and increasingly, agricultural and urban areas are further contributors to runoff generating surfaces.

Observational evidence of the spatial heterogeneity in surface moisture content is documented by Bell et al. (1980) and Owe et al. (1982) for small fields. The magnitude of the coefficient of variation of soil moisture content (which we will call cv) increases with the size of the field due to increased heterogeneity of topography and geology at larger scales.

As a first-order approximation to the spatial heterogeneity in surface moisture content, we take the point values within the large field to be distributed according to a two-parameter gamma pdf:

$$f_s(s) = \frac{\lambda^\alpha}{\Gamma(\alpha)} s^{\alpha-1} e^{-\lambda s}, \quad \lambda, \alpha, s > 0 \qquad (2)$$

where s is the surface layer point effective relative soil saturation defined by

$$s = \frac{\theta}{\theta_{sat}} = \frac{\theta}{n} \qquad (3)$$

where θ is the active (mobile) volumetric soil moisture content and n is the effective soil porosity. Bulk soil properties such as porosity are highly variable in space and have small correlation scales. For hydrologic processes averaged over the GCM grid scale, however, it is appropriate to assume that the soil profile is composed of homogeneous parallel layers with spatially averaged hydraulic properties.

At any time step and for every GCM land grid area, the grid mean surface layer relative soil saturation $E(s)$ is known and is propagated in successive GCM integrations as a model prognostic. The parameters of the Γ distribution are related, through the mean $E(s)$, according to

$$\lambda = \frac{\alpha}{E(s)}. \qquad (4)$$

The cv of the gamma distribution is

$$cv = \alpha^{-2}. \qquad (5)$$

Notice that we have not bounded the soil moisture pdf by the physical saturation value $s = 1$. The mass of the pdf above $s = 1$ mathematically represents the fraction of the grid area characterized by effective saturation. The dimensionless parameter α of the gamma pdf determines the shape of the distribution. The value $\alpha = 1$ represents the collapse of the gamma pdf to the exponential. With successively higher values of α (smaller cv's), the gamma pdf achieves greater central distribution. With the high cv's characteristic of the point distribution of surface relative soil saturation over large fields, the gamma pdf will possess a notable left skew. This is consistent with the physical situation in which, over entire watersheds, the surface soil moisture is laterally redistributed in a sloped terrain, and hence upland areas with the larger fraction of the total watershed area will experience soil saturations below the watershed mean. The relatively smaller areas in and around the drainage network or terrain foci will experience point relative soil saturations above the field mean, including some complete saturation.

d. Surface runoff

For a physically realistic parameterization of surface runoff, both the Horton and Dunne mechanisms of overland flow must be modeled. The two mechanisms of runoff generation may be summarized by:

surface runoff (q) contributors

$$= \text{Horton mechanism } (P - f^* \text{ for } P > f^* \text{ and } s < 1) + \text{Dunne mechanism } (P \text{ for } s \geqslant 1), \qquad (6)$$

where f^* is the infiltrability of the first soil layer. The first term on the right-hand side of (6) represents that portion of the point precipitation intensity that exceeds the infiltration rate into the soil. The second term refers to the precipitation that falls directly onto impermeable or saturated (ponded) surfaces. Where the first soil layer is saturated from below, the infiltrability is assumed to be zero. With this condition, and assuming independence of s and P, the general relationship for runoff rate (q) from the entire GCM grid during a time step is

$$\kappa^{-1}q = \int_0^1 \int_{f^*}^{\infty} (P - f^*) f_P(P) dP f_s(s) ds + \int_1^{\infty} \int_0^{\infty} P f_P(P) dP f_s(s) ds. \quad (7)$$

The two terms on the right-hand side correspond directly to the respective terms in (6).

After substituting (1) and (2) into (7) and integrating, we find the dimensionless runoff ratio $[R = q/E(P)]$ to be

$$R = \left[\frac{\alpha}{E(s)}\right]^{\alpha} \frac{1}{\Gamma(\alpha)} \int_0^1 s^{\alpha-1} e^{-\kappa f^*/E(P) - \alpha s/E(s)} ds + 1 - \frac{\gamma(\alpha, \alpha/E(s))}{\Gamma(\alpha)} \quad (8)$$

where $\gamma(\ ,\ )$ is the incomplete gamma function, and $\Gamma(\ )$ is the complete gamma function. The soil infiltrability f^* is dependent on the moisture conditions of the soil, i.e., $f^* = f^*(s)$. With the determination of the functional form of $f^*(s)$, the integration of (8) may be completed.

e. Infiltration rates

The general equation of vertical steady flow into unsaturated porous media is

$$q_z = K(s)\left[\frac{d\Psi}{dz} + 1\right] \quad (9)$$

where $K(s)$ is the unsaturated hydraulic conductivity of the soil at relative soil saturation s and Ψ is the matric potential. The terms inside the bracket represent the gradients of the capillary and gravity forces respectively in the vertical, z, direction. The capillary term dominates when soils are dry. This effect is more pronounced for finer textured soils. Moisture flow in coarse sandy soils, on the other hand, is largely gravitational.

With soil saturation at the very surface due to ponding, the vertical flow into the underlying unsaturated first soil layer is, after (9) and for a flat surface,

$$f^* = K(1)\left[\left.\frac{d\Psi}{dz}\right|_{\Psi=\Psi(1)} + 1\right] \quad (10)$$

where conductivity and capillary effects are evaluated at the interface of saturation. Applying the chain rule to (10),

$$f^* = K(1)\left[\left.\frac{d\Psi}{ds}\right|_{s=1} \frac{1-s}{-\Delta z} + 1\right] \quad (11)$$

where Δz is the thickness of the top soil layer and $(d\Psi/ds)$ is the slope of the so-called "moisture-retention curve" for soils. The moisture-retention curves do not have theoretical derivations, and they exhibit hysteresis which complicates their empirical determination. Experimental data on repeated draining and resaturating of soil columns, however, yield numbers that are fitted with simple curves [see appendix A, (A8) to (A10)].

We may write (11) in a more compact manner as

$$f^* = K(1)vs + K(1)(1 - v) \quad (12)$$

where

$$v = \left.\frac{d\Psi}{ds}\right|_{s=1} \frac{1}{\Delta z}. \quad (13)$$

f. The runoff ratio function

Equation (8) may be combined with (12) to yield a general expression for the runoff ratio under conditions of distributed moisture and precipitation intensity. We define the dimensionless saturated hydraulic conductivity ratio,

$$I = K(1)/E(P) \quad (14)$$

in order to write the runoff ratio as

$$R = 1 - \frac{\gamma(\alpha, \alpha/E(s))}{\Gamma(\alpha)} + \frac{e^{-\kappa I(1-v)}\gamma\left(\alpha, \kappa I v + \frac{\alpha}{E(s)}\right)}{\left(\frac{\kappa I v E(s)}{\alpha} + 1\right)^{\alpha}\Gamma(\alpha)}. \quad (15)$$

This is the result of convolving the two independent distributions of soil moisture and precipitation with a kernel that represents the physical equation of moisture infiltration into partially saturated soils. The expression further depends on the soils' capillary properties under conditions of less-than-full saturation.

To perform diagnostic studies of this runoff relationship, we take the simple case of gravitational flow only [$v \to 0$ in (12)] in which case

$$f^* \approx K(1) \quad (16)$$

and (15) becomes

$$R = \left[1 - \frac{\gamma(\alpha, \alpha/E(s))}{\Gamma(\alpha)}\right] + \left[\frac{\gamma(\alpha, \alpha/E(s))}{\Gamma(\alpha)}\right]e^{-\kappa I}. \quad (17)$$

The interpretation is now simple. A saturated fraction $\{1 - [\gamma(\alpha, \alpha/E(s))/\Gamma(\alpha)]\}$, representing the integral of the probability density function $f_s(s)$ above 1, has runoff ratio unity. Over this saturated fraction, all precipitation is runoff. The remaining or unsaturated fraction, $\{\gamma[\alpha, \alpha/E(s)]/\Gamma(\alpha)\}$ has runoff ratio

$e^{-\kappa I}$. As the wetted fraction κ becomes smaller, as the precipitation becomes more intense, or the saturated hydraulic conductivity becomes smaller, then the unsaturated fraction will have a higher runoff ratio. On the other hand, as ratio I grows, that is, as soils become exceedingly permeable, the contribution to the runoff ratio from the unsaturated fraction diminishes.

In either (15) or (17), the Dunne runoff $\{1 - [\gamma(\alpha, \alpha/E(s))/\Gamma(\alpha)]\}$ is the lower limit to the runoff ratio. Any contribution to the runoff ratio in excess of this lower limit is due to interactions of the precipitation intensity and the soil's moisture condition, i.e., Horton runoff.

g. The relative role of runoff types

The nonvanishing lower limit to the runoff ratio for moist soils is a distinct improvement over current parameterizations. For most soils and typical GCM grid precipitation intensities, the ratio I is expectedly large. Under nondistributed conditions, with the hydraulic conductivity of soils larger than the precipitation intensity, runoff is unlikely. With the assumed distributions and the physically realistic equations of moisture flow, however, significant runoff is possible even for large I.

We begin analyzing the behavior of the runoff ratio by assuming a fixed value of unity for the cv associated with the point distribution of the first soil layer relative saturation. Figure 2 shows the runoff ratio as a function of the GCM grid mean relative saturation for negligible soil capillarity, for $\alpha = 1$, and for the 60% fractional wetting which is consistent with observations for mesoscale rainfall (Eagleson 1984; Eagleson et al. 1987). The multiple curves correspond to various values of the ratio I which in this example represents moisture flow under gravity only. The lower the excess of saturated hydraulic conductivity with respect to precipitation (i.e., the lower I), the greater the possible runoff. At the other extreme, however, as the soil becomes more and more conductive (i.e., greater I), the runoff rates reduce but conveniently never vanish when the soil is relatively moist. This is due to the presence of a fraction of saturated surfaces within the basin.

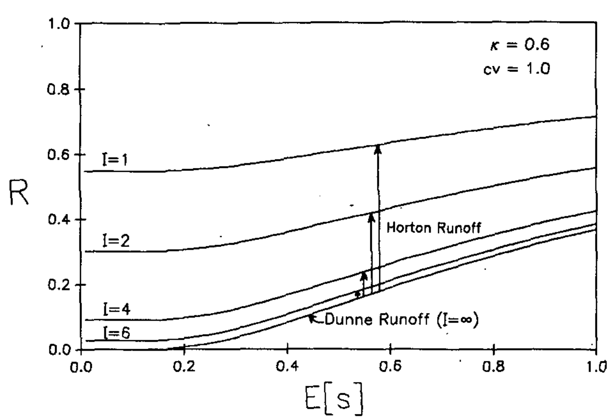

FIG. 2. Surface runoff ratio function for negligible soil capillarity, $\kappa = 0.6$, and $cv = 1.0$.

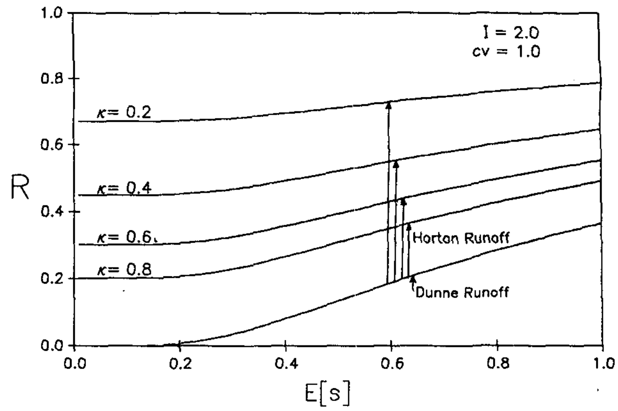

FIG. 3. Surface runoff ratio function for negligible soil capillarity, $I = 2.0$, and $cv = 1.0$.

Figure 3 also represents gravity-only infiltration, but here the ratio I is fixed at a lower value $I = 2$ and the wetted fraction κ is varied from 20% to 80%. The excess of runoff above the partial area-type lower limit may again be attributed to surface flow via the Horton mechanism. The smaller the wetted region, the higher is the precipitation intensity and thus the higher the Horton runoff rate.

Clearly the $\{1 - [\gamma(\alpha, \alpha/E(s))/\Gamma(\alpha)]\}$ lower limit to all runoff under any precipitation, soil moisture, soil type, and fractional wetting conditions refers to partial area-type (i.e., Dunne mechanism) surface runoff. Any runoff generated above this rate may be attributed to the Horton mechanism and is critically dependent on precipitation intensity. The existence of a distributed precipitation intensity field is thus an effective method of inducing surface runoff by the Horton mechanism as has been shown in another way by Milly and Eagleson (1988). However, this does not necessarily imply relative importance of the Horton mechanism as can be seen when typical soil properties are considered as in Fig. 4.

Figure 4a compares the runoff ratios for sandy loam soil (75% sand, 20% silt, and 5% clay) with [Eq. (15)] and without [Eq. (17)] the capillary effect and including the $I = \infty$ limit. For such soils the saturated hydraulic conductivity can be as high as tens and even hundreds of millimeters per hour; thus with typical precipitation intensities, the ratio I will be quite large. Notice for $I = 15$, the runoff ratio lies at the partial area-type lower limit. Therefore, for permeable soils, even with distributed precipitation intensities, the Horton mechanism is a minor contributor to total surface flow when compared with the Dunne mechanism.

For the heavier clay loam soil (30% sand, 35% silt, and 35% clay) the saturated hydraulic conductivity can

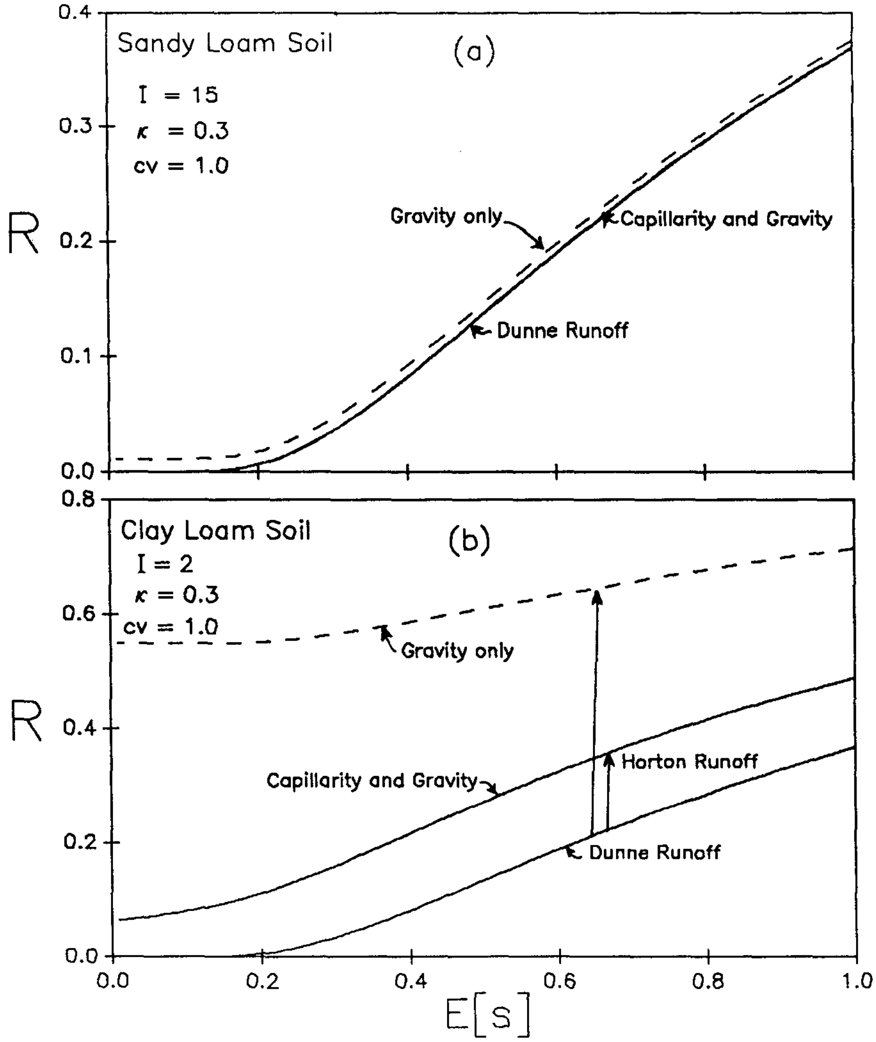

FIG. 4. Surface runoff ratio function with and without soil capillarity, $\kappa = 0.3$, $cv = 1.0$, depth of first soil layer = 0.1 m. (a) Sandy loam soil (75% sand, 20% silt, and 5% clay), and (b) clay loam soil (30% sand, 35% silt, and 35% clay).

be as low as a few millimeters per hour, and the ratio I may turn out to be as low as $I = 2$. For these soils the capillary effect is significant as can be seen in Fig. 4b which is plotted at half the vertical scale of Fig. 4a. For gravity only infiltration (Fig. 2) small I leads to large Horton runoff, but when capillarity is considered (Fig. 4b) even with small I, the Horton runoff is sharply reduced.

In summary, we see that for $\alpha = 1$ and over the normal range of saturated soil permeabilities, runoff is predominantly of the partial-area type except for clay soils where the Horton mechanism can play a significant role. This soil moisture parameterization is shown to be an improvement over those in current use in that it generates surface runoff when realistic values of soil conductivity and precipitation intensity are encountered. The distributed precipitation intensities over a fraction κ of the grid are shown to be important in allowing the possibility of surface runoff by the Horton mechanism.

h. *Effect of spatial variability on the runoff ratio*

Figure 5 illustrates the effect of the soil moisture distribution shape on the runoff ratio. In this figure $\kappa = 0.3$ and, for economy of presentation, only the $I = \infty$ case is considered. The shape of the soil moisture distribution is varied through changes in α (and therefore in the coefficient of variation of the underlying gamma distribution). The curves reflect the fraction of the GCM grid area that is saturated. This is equivalent to the probability mass concentrated above $s = 1$ for the particular gamma pdf. With $cv < 1$ and at lower mean soil saturations $[E(s) \ll 1]$, a lesser amount of

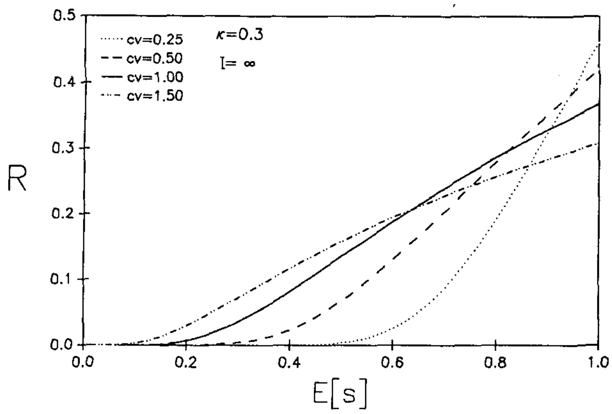

FIG. 5. The effect spatial variability of soil moisture on the runoff ratio function with $\kappa = 0.3$, and $I = \infty$.

the gamma pdf mass falls above $s = 1$ when compared to distributions with higher cv and the same $E(s)$. In the limit $cv = 0$ the soil moisture is uniform everywhere and there will be runoff in the $I = \infty$ case only with $E(s) = 1$ at which point $R = 1$. Furthermore, unsaturated soils with $cv = 0$ experience no surface runoff if $I \leqslant 1$ and the duration of the storm event is short.

3. Soil moisture losses due to evapotranspiration

Evapotranspiration is conveniently normalized by its limiting value, the atmospheric vapor transport capacity (i.e., potential evapotranspiration) e_p to obtain what is known as the evapotranspiration efficiency. This efficiency is currently parameterized as an empirical function of soil moisture in most GCMs (e.g., Sud and Fennessy 1982). In this section, a derived distribution of the evapotranspiration efficiency is obtained utilizing the basic soil moisture physics and an assumed heterogeneity of the process.

a. Subgrid heterogeneity of soil hydraulic conditions

Following the satisfaction of the canopy surface retention and surface runoff requirements, the residual precipitation depth is added to storage in the soil. Removal of the accumulation on the canopy and in the soil is then forced by the atmospheric evaporative demand. When the canopy surface is clear of retention depth, the plant may transpire and thereby remove soil moisture from the rooted soil layers.

Both bare soil evaporation and root soil moisture extraction processes are strongly dependent on the hydraulic state of the soil. The hydraulic state is in turn a function of soil type and saturation level. In section 2c, the soil moisture content of the first soil layer is assumed to be spatially distributed over the GCM grid [Eq. (2)]. There, the tail of the pdf above the soil saturation value represents the fraction of the GCM grid area that is characterized by soil saturation. In the evapotranspiration parameterization, a similar distribution is assumed for the relative saturation of the first soil layer.

b. Evaporation from bare soils

Under steady atmospheric forcing the loss of moisture from soil storage may cause a shift in the control of the evaporation rate from bare soils. When wet, a given soil may be capable of delivering moisture from soil storage to the surface at a rate equal to or greater than the atmospheric vapor transport capacity. Such cases are termed "climate-controlled" or "energy-limited" (Eagleson 1978c) as the actual evaporation rate will equal the climate-determined potential rate.

Continuing depletion of the stored soil moisture decreases the rate at which the soil can deliver moisture to the surface. At some moisture state, assuming constant climatic conditions, the rate of soil moisture delivery falls below the potential evaporation rate. At this and lower moisture states, the evaporation rate is then termed "soil-controlled" or "water-limited" (Eagleson 1978c) and is a nonlinear function of the moisture content due to its dependence on the soil water diffusivity. The relative soil saturation s^* at which the limits to evaporation shift is a function of the potential evaporation rate and of the soil properties.

c. Soil moisture desorption processes

In parameterizing the bare soil evaporation, we begin with the basic partial differential equation describing vertical moisture diffusion and moisture mass conservation in porous media,

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}\left[D(\theta)\frac{\partial \theta}{\partial z}\right] + \frac{\partial K}{\partial z} \tag{18}$$

where $D(\theta)$ is the soil water diffusivity and is defined by

$$D(\theta) = K(s)\frac{\partial \Psi}{\partial \theta}. \tag{19}$$

The initial and boundary conditions are taken as

$$\left.\begin{aligned} s(z, 0) &= \theta_0/\theta_{\rm sat} = s_0 \\ s(0, t) &= \theta_1/\theta_{\rm sat} = s_1 \end{aligned}\right\} \quad z, t \geqslant 0. \tag{20}$$

That is, a uniform moisture content s_0 characterizes the entire affected profile at time zero but a steady s_1 condition holds at the plane $z = 0$.

Philip (1957a, b) provides an approximate solution to (18) subject to (20) in which the desorption rate is proportional to $t^{-1/2}$. The constant of proportionality is referred to as the sorptivity S_e. The exfiltration rate under the combined influence of gravity and capillarity is given by

$$f_e = \frac{1}{2}S_e t^{-1/2} - \frac{[K(s_0) + K(s_1)]}{2} \tag{21}$$

and with $s_1 \ll s_0$,

$$f_e \approx \frac{1}{2} S_e t^{-1/2} - \frac{1}{2} K(s_0). \quad (22)$$

This equation represents the rate at which a uniformly wetted, semi-infinite porous medium loses moisture to vertical (upward) desorption. The drying of the uniformly wetted profile proceeds at a rate inversely proportional to the square root of time. Time scales associated with the substantial drying of soils are, however, generally larger than those used in integrating climate simulation models. This allows the time-averaging of the desorption rate over the GCM time step. At the end of each time step, the diagnostic variables such as the soil layer moisture contents are updated. At the beginning of the next time step, therefore, the desorption rate acts on an updated s_0 and $t = 0$ will again govern. The depth of the discretized (top) soil layer must be larger than the depth of moisture extraction during the time step in order for the semi-infinite initial condition to be valid.

The time-averaged desorption rate over the integration time T (GCM time step, e.g., one hour) is

$$\bar{f}_e = \frac{1}{T} \int_0^T f_e(t) dt$$

$$= S_e T^{-1/2} - \frac{1}{2} K(s_0). \quad (23)$$

The functional form of the desorptivity function S_e is derived in appendix A.

d. Bare soil evaporation under soil-controlled conditions

The time-averaged desorption rate is a function of the moisture content and of the unsaturated hydraulic conductivity. The rate of vertical moisture desorption from the top soil layer may be defined by combining (A9), (A14), and (23) into

$$\bar{f}_e = K(1)\Omega s_0^{1/2m+2} - \delta \frac{1}{2} K(1) s_0^{2/m+3}. \quad (24)$$

where Ω is a dimensionless soil parameter defined as

$$\Omega = \left[\frac{8n\Psi(1)}{3K(1)T(1+3m)(1+4m)}\right]^{1/2}. \quad (25)$$

The variable δ is a toggle which is either zero or one depending on whether the gravity term is to be excluded or included. Equation (24) is the time-averaged flux rate dependent on the initial relative soil saturation s_0 of the surface layer.

e. Derived distribution of bare soil evaporation efficiency

The evaporation rate is given by (24) whenever $\bar{f}_e$ is less than the potential evaporation rate e_p. Otherwise e_p is the governing loss rate. We let the relative soil saturation of the upper soil layer have the value s^* at the transition of evaporation rate control (i.e., $\bar{f}_e|_{s=s^*} = e_p$), i.e.,

$$\left.\begin{array}{ll} s \geq s^* & \text{climate-controlled evaporation} \\ s < s^* & \text{soil-controlled evaporation} \end{array}\right\}. \quad (26)$$

We combine these definitions and the spatial distribution of soil hydraulic states to write the spatial average bare soil evaporation from the GCM grid area as

$$E(\bar{f}_e) = e_P \int_{s^*}^{\infty} f_s(s) ds + \int_0^{s^*} \bar{f}_e f_s(s) ds. \quad (27)$$

1) GRAVITY-NEGLECTED CASE

Beginning with the simpler case that neglects the gravity term ($\delta = 0$), the substitution of (24) into (27) results in the definition of the bare soil evaporation efficiency (ratio of actual to potential evaporation),

$$\beta_s = 1 - \frac{\gamma(\alpha, \alpha\mathcal{E}^{-1})}{\Gamma(\alpha)} + (\alpha\mathcal{E}^{-1})^{-1/2m-2} \frac{\gamma\left(\frac{1}{2m} + 2 + \alpha, \alpha\mathcal{E}^{-1}\right)}{\Gamma(\alpha)} \quad (28)$$

where the dimensionless parameter $\mathcal{E}$ is simply

$$\mathcal{E} = E(s)/s^*. \quad (29)$$

The transitional relative soil saturation s^* is defined by equating e_p and $\bar{f}_e$ and solving for $s = s^*$. This leads to

$$\mathcal{E} = E(s)\left[\frac{K(1)\Omega}{e_p}\right]^{2m/1+4m}. \quad (30)$$

As with the case of runoff, we first assume $cv = 1$ (i.e., exponential pdf) in order to study the behavior of the evaporation efficiency function with respect to the soil type and climate variables.

The bare soil evaporation efficiency (gravity-neglected) is plotted in Fig. 6 for various soil types. From the definition of $\mathcal{E}$ we see that the evaporation efficiency is significantly increased as s^* decreases, and the climate, rather than the soil, becomes the controlling factor in determining the moisture loss from soil storage. We also note that other than through $\mathcal{E}$, the bare soil evaporation efficiency is only weakly dependent on soil type. The ratio $\mathcal{E}$ itself is strongly dependent on soil type even when similar atmospheric evaporative demands are imposed (Fig. 7).

2) GRAVITY-INCLUDED CASE

When the soil is rather moist and/or the soil is characterized by a high hydraulic conductivity, the gravity

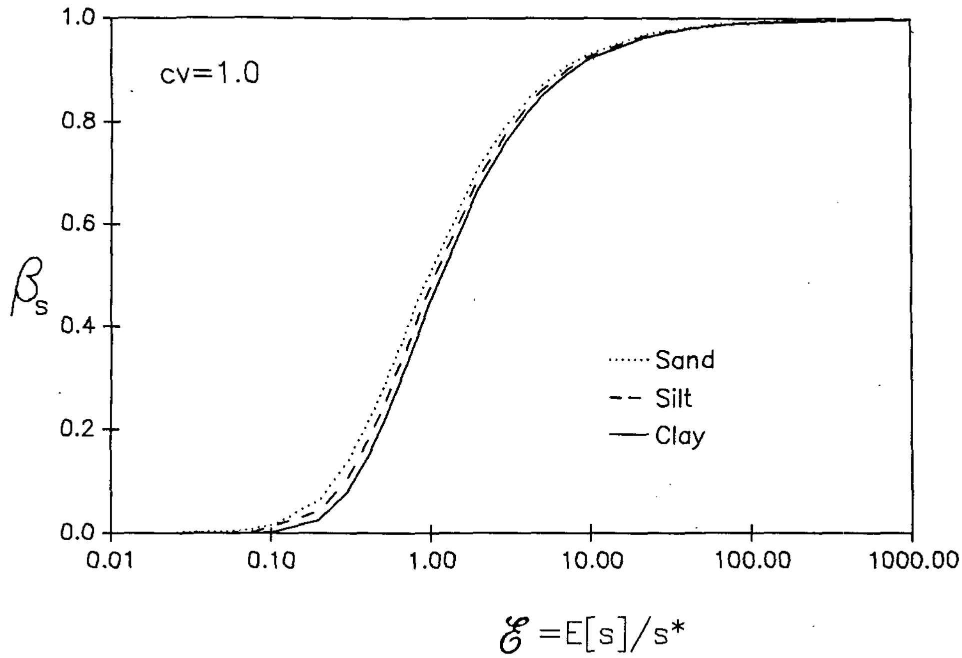

FIG. 6. Bare soil evaporation efficiency, cv = 1.0, and gravity neglected.

term may be a significant fraction of the desorption term. Under these circumstances, we take the full form of (24) with $\delta = 1$ and substitute it into (27) to derive the gravity-included bare soil evaporation efficiency,

$$\beta_s = 1 - \frac{\gamma(\alpha, \alpha\mathscr{E}^{-1})}{\Gamma(\alpha)} + \frac{K(1)}{e_p} \frac{\Omega(\alpha^{-1}\mathscr{E})^{1/2m+2}\gamma\left(\frac{1}{2m} + 2 + \alpha, \alpha\mathscr{E}^{-1}\right) - \frac{1}{2}(\alpha^{-1}\mathscr{E})^{1/2m+3}\gamma\left(\frac{1}{2m} + 3 + \alpha, \alpha\mathscr{E}^{-1}\right)}{\Gamma(\alpha)}. \quad (31)$$

Again the transitional relative soil saturation s^* is defined through equating e_p and $\bar{f}_e$, i.e.,

$$e_p = K(1)\Omega s^{*\,1/2m+2} - \frac{1}{2}K(1)s^{*\,2/m+3}. \quad (32)$$

Eliminate $K(1)/e_p$ between (31) and (32) and write

$$\beta_s = \frac{\Omega'\gamma\left(\frac{1}{2m} + 2 + \alpha, \alpha\mathscr{E}^{-1}\right) - \frac{1}{2}\gamma\left(\frac{2}{m} + 3 + \alpha, \alpha\mathscr{E}^{-1}\right)}{\left[\Omega'(\alpha\mathscr{E}^{-1})^{1/2m+2} - \frac{1}{2}(\alpha\mathscr{E}^{-1})^{2/m+3}\right]\Gamma(\alpha)} + 1 - \frac{\gamma(\alpha, \alpha\mathscr{E}^{-1})}{\Gamma(\alpha)} \quad (33)$$

where

$$\Omega' = \Omega\left[\frac{\alpha}{E(s)}\right]^{3/2m+1}. \quad (34)$$

3) RELATIVE IMPORTANCE OF GRAVITY

Figure 7 illustrates values for the transitional soil relative saturation s^* under a variety of soil and climatic conditions. The gravity-inclusive and -neglected Eqs. (30) and (32) are used to define s^*. In inspecting Fig. 7, we notice that over a wide range of soil and potential evaporation conditions, the gravity term does

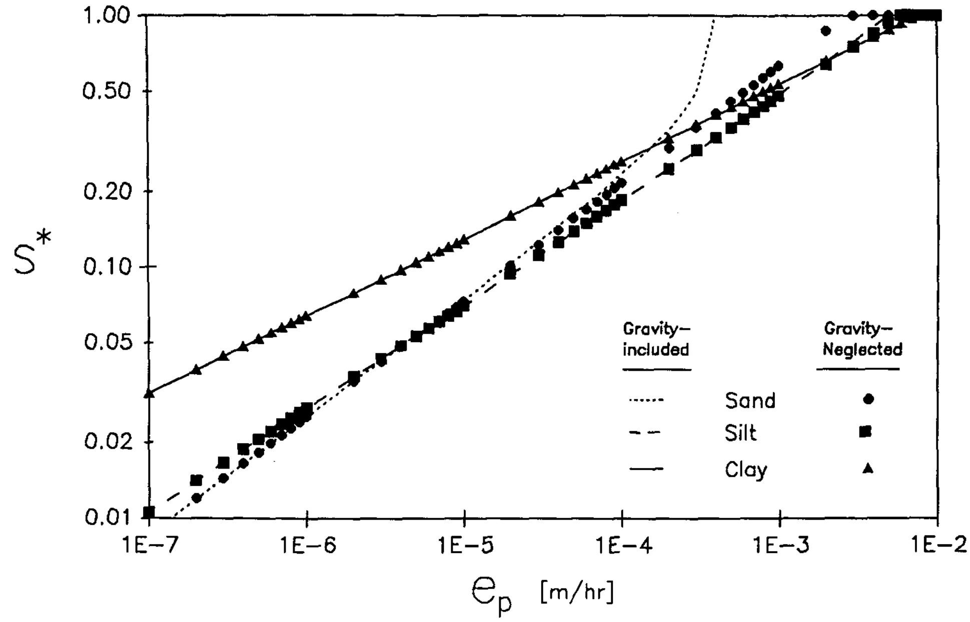

FIG. 7. The effect of gravity on the transitional relative soil saturation s^* as a function of the potential evaporation rate.

not significantly affect the resultant values of s^*. Thus the parameter $\mathscr{E}$ may be approximated as equivalent for both physical situations, in which case a rather simple expression may be presented relating bare soil evaporation efficiencies with and without the gravity term. Defining the dimensionless term

$$\Lambda = \frac{2\Omega' - \dfrac{\gamma\left(\dfrac{2}{m} + 3 + \alpha, \alpha\mathscr{E}^{-1}\right)}{\gamma\left(\dfrac{1}{2m} + 2 + \alpha, \alpha\mathscr{E}^{-1}\right)}}{2\Omega' - (\alpha\mathscr{E}^{-1})^{3/2m+1}} \quad (35)$$

allows us to write

$$[\beta_s]_{\text{gravity-included}} = \Lambda[\beta_s]_{\text{gravity-neglected}} + (1 - \Lambda)\left[1 - \frac{\gamma(\alpha, \alpha\mathscr{E}^{-1})}{\Gamma(\alpha)}\right]. \quad (36)$$

The parameter Λ represents the factor by which the water-limited evaporation efficiency is reduced due to the incorporation of gravity in the vertical soil moisture exfiltration formulation. Because gravity acts only to retard exfiltration out of the soil column, then $0 \leqslant \Lambda \leqslant 1$. With soil moisture subgrid spatial variability, a fraction $\{1 - [\gamma(\alpha, \alpha\mathscr{E}^{-1})/\Gamma(\alpha)]\}$ will have $s \geqslant s^*$ where the bare soil evaporation will be climate-controlled and independent of gravity. Reduction of the evaporation efficiency over both the water-limited and climate controlled regimes by Λ is the cause for the appearance of the compensating term $(1 - \Lambda)\{1 - [\gamma(\alpha, \alpha\mathscr{E}^{-1})/\Gamma(\alpha)]\}$ on the right-hand side of (36).

In Fig. 8 the linear *gravity-effect* term Λ is plotted for various soil and climatic conditions. As Λ approaches unity, the gravity term diminishes in significance when compared to the desorptive term [Eq. (36)]. The gravity term is less important for fine-textured soils and thus the values of Λ are practically equal to unity for silt and clay under all conditions. Only sandy soils wetted to near saturation exhibit slight reduction (mostly <20%) in evaporation efficiency due to the inclusion of the gravity term. When transitional relative soil saturation s^* is low, the bare soil evaporation over much of the GCM grid is climate-controlled. In this case and irrespective of soil type, the gravity term does not contribute significantly to the GCM grid evaporation efficiency.

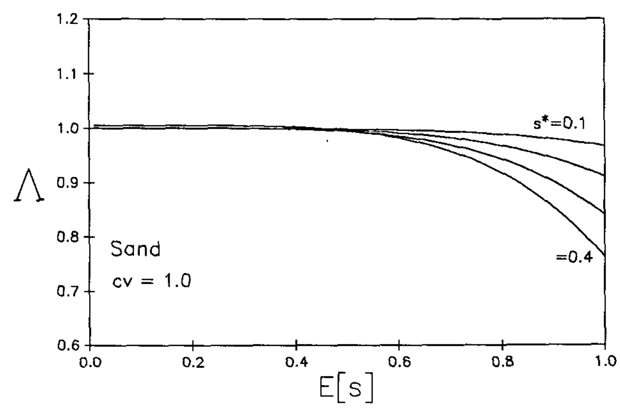

FIG. 8. Reduction in bare soil evaporation efficiency due to the inclusion of gravity.

f. Soil-water extraction by plant roots

Over the vegetated fraction of the GCM grid landsurface, the plant roots are active in releasing soil moisture into the atmosphere through transpiration. Transpiration is only possible when the canopy-intercepted moisture has evaporated.

The process of moisture extraction by roots escapes simple treatment. Molz (1981) catalogues various common approaches to bulk parameterization. Among the models that incorporate atmospheric, soil, and vegetation factors, the approach of Feddes et al. (1978) [also cited in Molz (1981)] is parsimonious and nontrivial. In this parameterization, for any single layer, the soil moisture extraction function $e_v(s)$ by roots is

$$e_v(s) = \begin{cases} 0, & 0 \leqslant s \leqslant s_w \\ e_p \dfrac{s - s_w}{s^* - s_w}, & s_w < s < s^* \\ e_p, & s^* \leqslant s < 1 \\ 0, & \text{saturated} \end{cases} \qquad (37)$$

where s_w is the relative soil moisture state below which the plant shuts its stomata and eventually wilts and s^* is that above which the transpiration by plants is limited by the evaporative demand of the atmosphere. We arbitrarily take this latter transition point above which transpiration is climate-controlled and below which transpiration is stomatal and soil-controlled, to be identical to that defined for bare soil evaporation. The gravity-neglected desorption is used. This is obviously only a practical approximation of a process which would otherwise require detailed consideration of microphysical processes in and around plant membranes embedded in porous media. For intermittently flooded regions where our unlimited soil moisture variable takes on the analytical state $s > 1$, the loss of soil moisture to the atmosphere by terrestrial plant root extraction ceases. However, direct evaporation at the potential rate from the canopy and surface is possible and will occur at the potential rate. For simplicity we also assume that the aquatic vegetation of perennial wetlands transpire at the potential rate e_p. Thus over the vegetated fraction of the GCM grid landsurface area, a statement of moisture loss by evapotranspiration will effectively include the plant root extraction relations of (37) with the provision of potential evaporation rates for all $s > s^*$.

The second transition point in the soil relative saturation is the wilting level s_w. Both soil and vegetation characteristics contribute to the definition of this point. For various vegetation types, the wilting matric potential is generally assumed known and constant. This level is translated into s_w by accounting for the soil hydraulic characteristics. Working with (A8), s_w is defined as

$$s_w = \left[\frac{\Psi_{\text{wilt}}}{\Psi(1)}\right]^{-m}. \qquad (38)$$

In the following derivations of a transpiration efficiency function, we assume independent distributed conditions for the topmost soil layer only. The remaining soil layers will have parameterizations that are discussed in the next section.

Using the gamma distribution of soil moisture in the first soil layer, the spatial average transpiration over the vegetated fraction is

$$E[e_v] = \int_{s_w}^{s^*} e_v(s) f_s(s) ds + e_p \int_{s^*}^{\infty} f_s(s) ds. \qquad (39)$$

Substituting (30) and (37) into (39) and integrating, we have the spatial average transpiration efficiency

$$\beta_v = \frac{E[e_v(s)]}{e_p} = 1 + \frac{\gamma(\alpha + 1, \alpha\mathcal{E}^{-1}) - \alpha\mathcal{E}^{-1}\gamma(\alpha, \alpha\mathcal{E}^{-1}) - \gamma(\alpha + 1, \alpha\mathcal{W}^{-1}) + \alpha\mathcal{W}^{-1}\gamma(\alpha, \alpha\mathcal{W}^{-1})}{\Gamma(\alpha)(\alpha\mathcal{E}^{-1} - \alpha\mathcal{W}^{-1})} \qquad (40)$$

where

$$\mathcal{W} = \frac{E(s)}{s_w}. \qquad (41)$$

This equation represents the reduction of transpiration below the potential value due to soil and vegetation factors under conditions of independently distributed soil moisture over the GCM grid area. Above relative saturation equal to the combined soil–atmosphere parameter s^*, transpiration proceeds at its potential rate. Below the wilting level s_w, the root extraction of moisture (and hence transpiration), is identically zero. And for relative soil saturations between s_w and s^*, the transpiration efficiency rises linearly from zero to unity. Unlike the bare soil case, the lower bound to s^* is s_w. The upper bound is unity. In fact, when $s^* \to s_w(\mathcal{E} \to \mathcal{W})$ or $s_w \geqslant s^*(\mathcal{W} \leqslant \mathcal{E})$, β_v reaches a limit that is only visible after applying L'Hôpital's rule to (40):

$$\lim_{s^* \to s_w} \beta_v = 1 - \frac{\gamma(\alpha, \alpha\mathcal{W}^{-1})}{\Gamma(\alpha)}. \qquad (42)$$

In this special case (herbaceous crops and short grasses, for example) the transpiration efficiency is unity for the fraction of the grid area that has soil relative saturation above wilting under the defined spatial distribution.

In Fig. 9, β_v is plotted against the parameter $\mathcal{E}$ for the two limits $s_w = 0$ and $s_w = s^*$ with $\alpha = 1$. In sandy soils, setting $s_w = 0$ would be reasonable since, over the typical range of values for Ψ_{wilt}, the relative soil saturation at the wilting point generally stays close to zero. In heavier soils, however, strong matric potentials are present even with high relative saturations. There, s_w may be the limit to s^* and thus (42) the limit to (40).

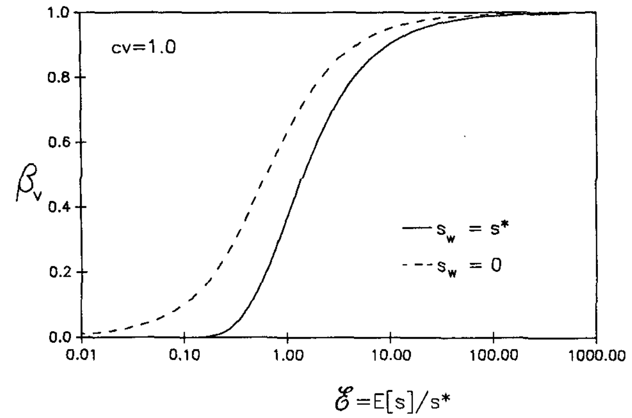

FIG. 9. Transpiration efficiency with cv = 1.0.

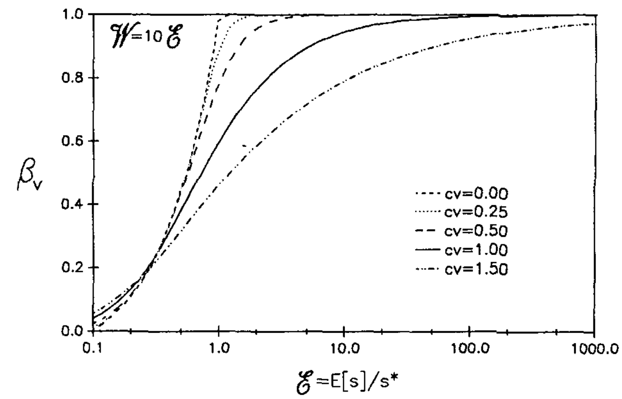

FIG. 11. Sensitivity of the transpiration efficiency function to spatial variability of soil moisture with $\mathscr{W} = 10\mathscr{E}$ or $s_w = 0.1s^*$.

g. Effect of spatial variability of soil moisture on the bare soil evaporation and the transpiration efficiencies

In Figs. 10 and 11, β_s and β_v are plotted against $\mathscr{E} = E(s)/s^*$ for $m = 1$ and cv values 0.25, 0.5, 1.0, and 1.5. Also plotted are the curves for the $cv = 0$ limit in which case the soil moisture is uniform everywhere over the GCM grid area. In the vicinity of $\mathscr{E} = 1$, we see that β is sensitive to cv. Again lower cv's imply greater central distribution and less variance about the mean for the gamma pdf of the surface moisture conditions. With higher cv's, the probability mass of the gamma pdf has greater dispersion about its mean $E(s)/\alpha$.

For $\mathscr{E} \gg 1$, β's are large because the average soil moisture condition exceeds that for which the climate controls the evaporation or transpiration. Under this condition increasing cv means more of the soil will be under soil control and hence the values of the β's decline.

For $\mathscr{E} \ll 1$, on the other hand, the average soil moisture condition is less than that for which the climate controls the evaporation. Under this condition increasing cv means more of the soil will be under climate control and hence the values of the β's rise. This reversal behavior occurs where the median s equals s^* which is at $\mathscr{E} > 1$ due to the characteristic skew of the gamma pdf.

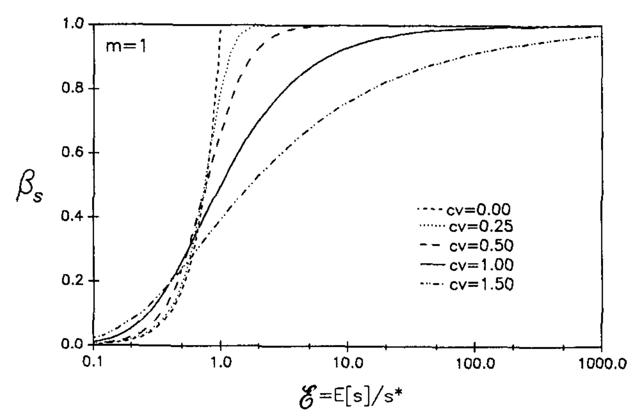

FIG. 10. Sensitivity of the bare soil evaporation efficiency to spatial variability of soil moisture with m = 1.0.

h. Extension of the transpiration efficiency to multiple soil layers

Of the three hydrologic processes considered so far, surface runoff, bare soil evaporation, and soil moisture extraction by plant roots, only the last may have direct influence on the deeper soil horizons. Surface runoff generation and the drying of exposed soil surfaces are processes whose instantaneous magnitudes depend primarily on the near surface soil conditions. In this section, we present an extension of the transpiration efficiency function to include soil moisture extraction by plant roots from multiple soil layers.

In consideration of a multiple layer soil system, the subscripted indices on the relevant parameters will represent the characteristics of the respective soil level. The spatial heterogeneity within the large GCM grid area has again been modeled as a random process whose expectation is the mean grid condition. The realization of the process over all points in the GCM grid area follows a gamma distribution. Consider each such point to be the focus of a hydrologic subarea within the larger GCM grid area. In the parameterization of multiple soil layered systems, we assign a spatial probability density function to only one layer (e.g., topmost layer), hereafter referred to as the base-line process with subscript j. All other soil layers i ($i = 1, 2, \cdots, N$; $i \neq j$) have *no* probability distributions themselves, but their soil moisture states are dependent on the realization of the random process at the base-line level due to physical considerations. Within the same hydrologic subarea, we expect the soil moisture at all levels i ($i \neq j$) to be consistent with the base-line j-level process. We require that wet (dry) soil layers are not haphazardly stacked on dry (wet) adjacent layers within the same hydrologic subarea.

Though the soil moisture states at levels i ($i \neq j$) do not have explicit spatial probability distributions, we nevertheless require that they scale consistently with the realization of the baseline process in the manner

$$s_i = g(s_j) = \frac{s_j}{E(s_j)} E(s_i), \quad i = 1, 2, \cdots, N; \quad i \neq j. \tag{43}$$

We assume that the pdf shape parameter α is the same for all layers. This relation holds independently for all levels i when paired with the base-line level j process. The expectation of the soil moisture states over all hydrologic subareas at level i, as represented by (43), will be equal to the GCM grid-average value that is propagated in every model integration step. To define the transpiration efficiency for the i-th soil level, we produce the derived distribution of s_i given that the baseline process has a gamma spatial distribution. The basic step is to apply

$$f_{s_i}(s_i) = \left|\frac{\partial g^{-1}(s_i)}{\partial s_i}\right| f_{s_j}[g^{-1}(s_i)], \tag{44}$$

which when evaluated becomes

$$f_{s_i}(s_i) = \left[\frac{\alpha}{E(s_i)}\right]^{\alpha} \frac{s_i^{\alpha-1}}{\Gamma(\alpha)} e^{-\alpha s_i/E(s_i)}. \tag{45}$$

The result is a gamma derived distribution of the i-th level soil moisture point distribution in the hydrologic subareas with independent parameter $E(s_i)$. We may now proceed directly to the equations describing single-layer transpiration efficiency to evaluate the transpiration efficiency within a multilayered soil system. Introduce subscript i to denote the transpiration efficiency [Eq. (40)] evaluated for the independent parameters of the i-th soil layer with a fraction ϵ_i of the available roots. The total transpiration efficiency now becomes

$$\beta_v = \sum_{i=1}^{N} \epsilon_i \beta_{v_i} \tag{46}$$

with the choice of the baseline process (level j) being arbitrary.

4. Implementation

The practical use of the proposed subgrid parameterization of surface hydrologic processes described here requires the definition of several fixed parameters. Table 1 lists values that we consider to be reasonable.

The fractional wetting by rainfall events (κ) may be related to the relative size of the air mass parcel in GCMs that have moist convection parameterizations with variable plume fractions (Kuo-type). Remote sensing of variables such as outgoing longwave radiation may be used as a way to determine the climatology of convectively active areas within larger fields. Eagleson (1984) and Eagleson et al. (1985, 1987) demonstrate that generally about 60% of storm areas are actually wetted by rainfall. Depending on the ratio of the GCM grid area to typical storm areas, this value is proportionally scaled. For most GCMs in current use, typical storms cover about half of the grid area. We assume that large-scale condensation occurs over the entire storm area but convective precipitation covers 60% of the total storm area.

The cv of the subgrid distribution of surface soil moisture content may be intrinsically dependent on the grid mean value (Bell et al. 1980; Owe et al. 1982). It is strongly dependent on the area of the field used to compute the statistics of spatial heterogeneity. For areas on the order of 10^2 to 10^5 km^2, comparable to GCM grids, cv = 1 (exponential pdf) is a reasonable

TABLE 1. Parameters of the surface hydrologic subgrid parameterization.

Rainfall	κ (fractional wetting)	Storm area to GCM grid area fraction for large-scale condensation. For moist convection, reduce this value 60% or relate to parcel size.		
Soils	cv (coefficient of variation)	1.0 (exponential) or tie to grid mean		
	Δz (top layer thickness [m])	0.1		
		Sand	*Silt*	*Clay*
	m (soil pore-size distribution)	3.3	1.2	0.4
	$K(1)$ (saturated conductivity) [10^{-3} m h^{-1}]	7.5	2.2	0.3
	$\Psi(1)$ (saturated soil matric potential) [m]	0.23	0.46	0.93
	n (effective porosity)	0.25	0.35	0.45
Vegetation	Ψ_{wilt} (wilting matric potential) [10^2 kPa]	−15 for swamp plants and herbaceous crops; −15 to −25 for grasses; −15 to −25 for temperate zone woody species; −18 to −25 for conifers; and −55 to −90 for desert plants		

operational choice. With $\alpha = 1$, the analytical expressions are simplified considerably, thus reducing the computational burden. Again remote sensing of soil moisture on fields with various areas and in different climates may provide clues as to the choice of cv.

Representative values for the empirical parameters $K(1)$, $\Psi(1)$, and m used in characterizing the hydraulic properties of unsaturated soils are based on our interpretation of the case studies compiled by Mualem (1976, 1978). It should be noted that θ is the effective water content, i.e., the water content with the immobile residual water content subtracted. The variable s is thus the effective relative soil saturation. Furthermore, $\Psi(1)$, the "saturated" matric potential is a parameter of the fitted moisture retention curve.

The thickness of the first soil layer appears explicitly as a parameter in the runoff parameterization. Its presence is only implicit in the bare soil evaporation efficiency function. The value of Δz for the runoff case must be comparable in magnitude to the effective depth of infiltrating water during GCM integration steps. In the presence of soil capillarity, we take Δz to be near 10 cm. To be compatible with the assumption of a semi-infinite soil column in deriving the desorption rate equation, the effective depth of moisture extraction due to bare soil evaporation must be less than Δz. A depth of 10 cm satisfies this criterion for the given typical GCM time step.

The vegetation wilting point matric potential and fraction of active roots in different layers depends on the database used in the GCM ground hydrology submodel. In Table 1 we present some typical values for the former.

5. Concluding remarks

Expressions have been derived to describe the runoff ratio, bare soil evaporation efficiency, and transpiration efficiency from a GCM grid area including subgrid scale spatial variability. The soil moisture conditions and the precipitation intensity are assumed to be spatially distributed over the grid area according to gamma and exponential point distributions respectively whose parameters correspond to the areal mean values that are propagated in GCM integrations. The derivations have been based largely on dynamic hydrological relationships characterizing these processes.

The runoff ratio function is based on both the point-excess of rainfall intensity with respect to the local infiltrability, and the precipitation over the saturated area in near-channel zones and depressions. The essentials of soil moisture physics are retained in producing the runoff ratio which is a function of the point soil relative saturation, unsaturated soil hydraulic properties, and the precipitation intensity.

The bare soil evaporation efficiency is based on desorption physics for drying porous media. Soil capillarity and the strength of the atmospheric evaporative demand are incorporated into the model such that the transition from a water-limited to a climate-controlled evaporation regime is modeled. The derived distribution of bare soil evaporation efficiency over a grid area with subgrid variance in soil moisture is necessary since the moisture loss from the soil is a nonlinear function of moisture content. Such dependence implies that the evaporation for the average soil moisture condition is not necessarily the same as the average of the evaporative flux for the distributed soil moisture conditions.

A similar expression is derived for the transpiration efficiency. A simplified root soil moisture extraction model is assumed in this case. Extensions of the transpiration efficiency for a multiple layer soil system are established based on simple assumptions concerning the moisture states of soil layers for the same hydrologic subareas.

The GCM computations may be generally subdivided into dynamical and physical parameterizations. The latter often require the consideration of processes on scales smaller than those sufficient for dynamical computations. The landsurface hydrology, as it has been presented here, appears to contain processes that warrant subgrid parameterization. However, such a parameterization should be incorporated in GCMs only if it promises to improve the model climate or to increase the model capability in capturing the consequences of environmental change. The appropriate sensitivity tests and verification of the hydrologic parameterizations are tasks which remain for a follow-up study.

Subgrid parameterization within GCMs will always remain the Achilles' heel of numerical climate simulation. More machine resources will lead to finer mesh models, yet never on the spectrum of scales actually present in the real physical processes. Unresolved spatial heterogeneity in hydrologic processes result in variabilities that are not captured when the large grid area is assumed to be a uniform hydrologic unit. The statistical–dynamical approach to hydroclimatological modeling promises to be a practical method of approaching this problem.

Acknowledgments. This work was supported by the National Aeronautics and Space Administration under Grant NAG 5-743.

APPENDIX A

Desorptivity Function

Sorptivity is a soil capillary property and is independent of gravity. In defining the desorptivity S_e, we thus begin with the horizontal (x-direction) soil moisture diffusion equation

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x}\left[D(\theta)\frac{\partial \theta}{\partial x}\right]. \quad \text{(A1)}$$

For desorption boundary and initial conditions ($\theta(0,$

$t) = 0; \theta(x, 0) = \theta_0; x, t \geq 0)$, one may define a weighted *effective* diffusivity D_e such that (A1) becomes

$$\frac{\partial \theta}{\partial t} = D_e \frac{\partial^2 \theta}{\partial x^2} \quad \text{(A2)}$$

with the solution (Carslaw and Jaeger 1959)

$$\theta(x, t) = \theta_0 \operatorname{erf}\left[\frac{x}{2\sqrt{D_e t}}\right]. \quad \text{(A3)}$$

The flux across the plane $x = 0$, differentiating (A3), is

$$\text{flux} = D_e \frac{\partial \theta}{\partial x}\bigg|_{x=0} = \theta_0 \left[\frac{D_e}{t\pi}\right]^{1/2}. \quad \text{(A4)}$$

Eliminating $t^{-1/2}$ between Eqs. (A4) and (22) without the gravity terms ($\delta = 0$; horizontal desorption) results in

$$S_e = 2\theta_0 \left[\frac{D_e}{\pi}\right]^{1/2}. \quad \text{(A5)}$$

We parameterize effective diffusivity D_e similar to Eagleson (1978b)

$$D_e = \varphi_e D(\theta) \quad \text{(A6)}$$

where φ_e is the coefficient by which the soil water diffusivity is scaled in order to yield the *effective* diffusivity for the prescribed boundary and initial conditions. Combining (A5) and (A6) yields

$$S_e = 2\theta_0 \left[\frac{\varphi_e D(\theta_0)}{\pi}\right]^{1/2}. \quad \text{(A7)}$$

Once the functional forms of φ_e and soil water diffusivity $D(\theta)$ are defined, the expression for S_e dependent on the initial moisture content θ_0 is complete.

The soil hydraulic properties are parameterized analogous to Brooks and Corey (1966) as

$$\Psi(s) = \Psi(1) s^{-1/m} \quad \text{(A8)}$$

and

$$K(s) = K(1) s^{2/m+3} \quad \text{(A9)}$$

where m is the soil pore-size distribution index. From the definition of soil water diffusivity [Eq. (19)],

$$D(s) = K(s) \frac{d\Psi}{d\theta} = \left|\frac{K(1)\Psi(1)}{nm}\right| s^{1/m+2} \quad \text{(A10)}$$

where n is the effective porosity.

Recently Parlange et al. (1985) note that for desorption

$$S_e^2 = \frac{8}{3}\theta_0^2 \int_0^1 \left(1 - \frac{\theta}{\theta_0}\right) D\left(\frac{\theta}{\theta_0}\right) d\frac{\theta}{\theta_0} \quad \text{(A11)}$$

which upon integration with (A10) yields

$$S_e^2 = \frac{8\theta_0^2}{3\left(\frac{1}{m} + 3\right)\left(\frac{1}{m} + 4\right)} \left[\frac{K(1)\Psi(1)}{nm} s_0^{1/m+2}\right] = \frac{8\theta_0^2}{3\left(\frac{1}{m} + 3\right)\left(\frac{1}{m} + 4\right)} D(s_0) \quad \text{(A12)}$$

where $s_0 = \theta_0/\theta_{sat} = \theta_0/n$.

Combining (A7) and (A12) results in the definition

$$\varphi_e = \frac{2\pi m}{3(1 + 3m)(1 + 4m)}. \quad \text{(A13)}$$

To complete the definition of the desorptivity function, we back-substitute (A13) into (A7),

$$S_e = \left[\frac{8nK(1)\Psi(1)}{3(1 + 3m)(1 + 4m)}\right]^{1/2} s_0^{1/2m+2}. \quad \text{(A14)}$$

APPENDIX B

Symbols

cv	coefficient of variation (standard deviation/mean)
$D(\ )$	soil water diffusivity $[L^2T^{-1}]$
D_e	effective soil-water diffusivity $[L^2T^{-1}]$
$E(\ )$	spatial expectation operator
$e_v(\ )$	transpirational soil moisture loss $[LT^{-1}]$
e_p	potential evaporation $[LT^{-1}]$
$\mathcal{E}$	combined soil-climate parameter
erf( )	the error function
$f(\ )$	probability density function
f_e	exfiltration rate $[LT^{-1}]$
f^*	first soil layer infiltrability $[LT^{-1}]$
$g(\ )$	functional dependence of soil moisture on the base-line process
$K(\ )$	soil water hydraulic conductivity $[LT^{-1}]$
m	soil pore-size distribution index
N	number of soil layers
n	effective soil porosity
P	point precipitation rate $[LT^{-1}]$
q	runoff rate $[LT^{-1}]$
q_z	vertical flow of soil moisture $[LT^{-1}]$
R	runoff ratio to precipitation
S_e	desorptivity $[LT^{-1/2}]$
s	effective relative soil saturation
s_0	relative soil saturation at beginning of time step
s^*	transitional relative soil saturation
s_w	relative soil saturation at vegetation wilting point
t	time $[T]$
T	GCM integration period $[T]$
$\mathcal{W}$	ratio of $E(s)$ to s_w
x	horizontal coordinate $[L]$
z	vertical coordinate (positive upward) $[L]$

Δz	thickness of soil layer $[L]$
α	gamma pdf shape parameter
β_s	bare soil evaporation efficiency
β_v	vegetation transpiration efficiency
ϵ_i	fraction of roots in layer i
δ	toggle variable (0 or 1 state)
θ	effective soil water content
θ_{sat}	saturated soil moisture content
$\gamma(\ ,\ )$	incomplete gamma function
$\Gamma(\ )$	gamma function
κ	wetted fraction of grid area
λ	gamma pdf parameter
Λ	gravity term coefficient for evaporation efficiency
φ_e	coefficient for effective diffusivity
$\Psi(\ )$	soil matric potential $[L]$
Ψ_{wilt}	soil matric potential at plant wilting point $[L]$
Ω	combined soil parameter
$(\bar{\ })$	temporal average operator

REFERENCES

Abramopoulos, F., C. Rosenzweig and B. Choudhury, 1988: Improved ground hydrology calculations for GCMs—Soil water movement and evapotranspiration. *J. Climate,* **1**(9), 921–941.

Bell, K. R., B. J. Blanchard, T. J. Schmugge and M. W. Witczk, 1980: Analysis of surface moisture variations within large-field sites. *Water Resour. Res.,* **16**(4), 796–810.

Brooks, R. H., and A. T. Corey, 1966: Properties of porous media affecting fluid flow. *J. Irrig. Drain. American Soc. Civil Eng., IR* **2,** 61–88.

Carslaw, H. S., and J. C. Jaeger, 1959: *Conduction of Heat in Solids,* 2nd ed. Oxford University Press, 510 pp.

Carson, D. J., 1982: Current parameterizations of landsurface processes in atmospheric general circulation models. *Landsurface Processes in Atmospheric General Circulation Models,* P. S. Eagleson, Ed., Cambridge University Press, 67–108.

Dunne, T., and R. D. Black, 1970: Partial area contributions to storm runoff in a small New England watershed. *Water Resour. Res.,* **6**(5), 1296–1311.

Eagleson, P. S., 1978a: Climate, soil, and vegetation. Part 1: Introduction to water balance dynamics. *Water Resour. Res.,* **14**(5), 705–712.

——, 1978b: Climate, soil, and vegetation. Part 3: A simplified model of soil moisture movement in the liquid phase. *Water Resour. Res.,* **14**(5), 722–730.

——, 1978c: Climate, soil, and vegetation. Part 4: The expected value of annual evapotranspiration. *Water Resour. Res.,* **14**(5), 731–739.

——, 1984: The distribution of catchment coverage by stationary rainstorms. *Water Resour. Res.,* **20**(5), 581–590.

——, and Q. Wang, 1985: Moments of catchment storm area. *Water Resour. Res.,* **21**(8), 1185–1194.

——, N. M. Fennessey, Q. Wang and I. Rodriguez-Iturbe, 1987: Application of spatial Poisson models to air mass thunderstorm rainfall. *J. Geophys. Res.,* **92**(D8), 9661–9678.

Feddes, R. A., P. J. Kowalik and H. Zaradny, 1978: *Simulation of Field Water Use and Crop Yield.* John Wiley and Sons, 188 pp.

Freeze, R. A., 1974: Streamflow generation. *Water Resour. Res.,* **12**(4), 627–647.

Hewlett, J. D., 1961: Watershed management. Rep. for 1961 Southeastern Forest Experiment Station, Asheville, NC, 61–66. [Available from present authors.]

——, and R. A. Hibbert, 1965: Factors affecting the response of small watersheds to precipitation in humid areas. *Proc. International Symposium on Forest Hydrology,* Pennsylvania State University, University Park, PA 16 pp [Available from present authors.]

Koster, R. D., P. S. Eagleson and W. S. Broecker, 1988: Tracer water transport and subgrid precipitation variation within atmospheric general circulation models. Ralph M. Parsons Lab. Rep. #317, Department of Civil Engineering, Massachusetts Institute of Technology, 364 pp.

Milly, P. C. D., and P. S. Eagleson, 1988: Effects of storm scale on surface runoff volume. *Water Resour. Res.,* **24**(4), 620–624.

Molz, F. J., 1981: Models of water transport in the soil-plant system: A review. *Water Resour. Res.,* **17**(5), 1245–1260.

Mualem, Y., 1976: A catalogue of hydraulic properties of unsaturated soils. Hydrodynamics and Hydraulics Lab., Israel Institute of Technology, 100 pp. [Available from U.S. Department of Agriculture, Beltsville, MD.]

——, 1978: Hydraulic conductivity of unsaturated porous media: Generalized macroscopic approach. *Water Resour. Res.,* **14**(2), 325–334.

Owe, M., E. B. Jones and T. J. Schmugge, 1982: Soil moisture variation patterns observed in Hand County, South Dakota. *Water Resour. Bull.,* **18**(6), 949–954.

Parlange, J-Y., M. Vauclin, R. Haverkamp and I. Lisle, 1985: Note: The relation between desorptivity and soil-water diffusivity. *Soil Science,* **139,** 458–461.

Philip, J. R., 1957a: The theory of infiltration. Part 1: The infiltration equation and its solution. *Soil Science,* **83,** 345–357.

——, 1957b: The theory of infiltration. Part 4: Sorptivity and algebraic infiltration equations. *Soil Science,* **84,** 257–264.

Sud, Y. C., and M. S. Fennessy, 1982: An observational-data-based evapotranspiration function for General Circulation Models. *Atmos. Ocean,* **20,** 301–316.

Warrilow, D. A., A. B. Sangster and A. Slingo, 1986: Modelling of land surface processes and their influence on European climate. U.K. Meteorological Office, DCTN 38, 94 pp. [Available from the UK Meteorological Office.]