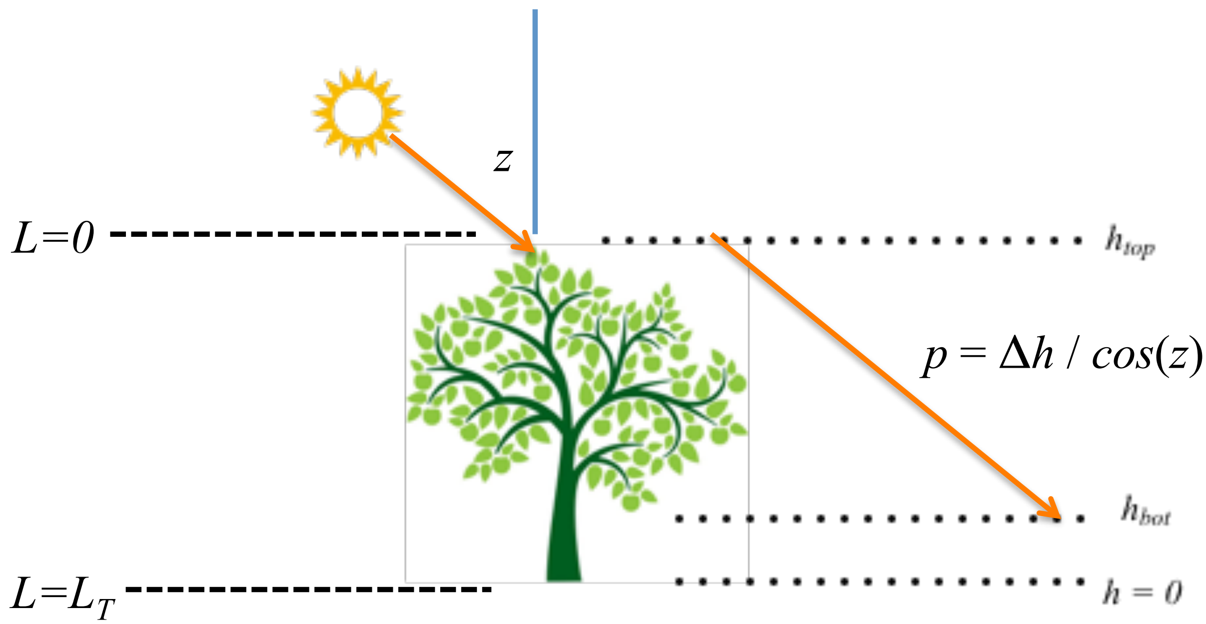
Leaf-to-Canopy Scaling following Sellers et al 1992



Consider a canopy of height h_{top} as shown above.

Define a vertical coordinate L corresponding to the leaf area index above height h. In other words, L is the ratio of the area of leaves above height h to the area of the ground.

Let $PAR(L)$ be the photosynthetically-active radiation at level L in the canopy and assume that this drops off exponentially in the direction of the solar beam according to Beer's Law

$$PAR(L) = PAR_0 e^{-kL} \quad [1]$$

where PAR_0 is the incident radiation at the top of the canopy, k is the extinction coefficient for *PAR* in the direction of the solar beam (along the path length $p = (h_{top} - h_{bot}) / cos(z)$ in the diagram above), and z is the solar zenith angle. The extinction coefficient is the projected leaf area index in the direction of z, and is written as $k = G(\mu)/\mu$ where $\mu = cos(z)$ is the cosine of the solar zenith angle.

From equation [1], the radiation at the ground surface $h = 0$ is $PAR(L_T) = PAR_0 e^{-kL_T}$. The difference between the incident radiation PAR_0 and the radiation reaching the ground is the *absorbed PAR*:

$$APAR = PAR_0 - PAR_{L_T} = PAR_0 - PAR_0 e^{-kL_T} = PAR_0(1 - e^{-kL_T}). \quad [2]$$

From equation [2], fraction of PAR absorbed by the canopy is just

$$FPAR = APAR/PAR_0 = 1 - e^{-kL_T}. \qquad [3]$$

Sellers et al (1992) assumed that nitrogen allocation (Vmax) also followed the time-averaged light profile down though the canopy, and showed that in this case

$$GPP = \int_0^{L_T} GPP(L)dL = GPP_{L=0}\int_0^{L_T} e^{-kL}dL \qquad [4]$$

The definite integral in equation 4 can be written as

$$GPP = GPP_{L=0}\frac{-(e^{-kL_T} - e^0)}{k} = GPP_{L=0}\left[\frac{-(e^{-kL_T} - 1)}{k}\right] = GPP_{L=0}\Pi$$

where $GPP_{L=0}$ is the photosynthesis rate of a fully-developed and illuminated "sun leaf" at the top of the canopy, and

$$\Pi = \frac{1 - e^{-kL_T}}{k} \qquad [5]$$

is a canopy scaling factor that relates top-leaf to total canopy photosynthesis.

Substituting from equation [3] into equation [5], $\Pi = \frac{FPAR}{k}$.

In SiB, we call this canopy scaling factor APARKK, so that

total GPP = GPP(sun-leaf) * APARKK.