Blackbody Radiation

CHAPTER 2

How light can carry heat energy through empty space

Heat

Heat is simply the bouncing-around energy of atoms. Atoms in gases and liquids fly around, faster if it is hot and slower if it is cold. Atoms with chemical bonds between them stretch, compress, and bend those bonds, vibrating more energetically at high temperatures.

Perhaps you knew this already, but have you ever wondered how you can feel such a thing? Atoms are tiny things. We cannot really feel individual atoms. But it doesn't take any state-of-the-art laboratory technology to tell how fast the iron atoms in your stove burner are bouncing around. All you have to do is touch it. Actually, another method may be occurring to you—that you could look at it; if it is hot, it will glow. It glows with blackbody radiation, which we shall return to later.

A thermometer is like an atomic speedometer.

You can feel the hot stove because the energetic "bounciness" of the stove atoms gets transferred to the atoms in your nerves in your fingers. The fast-moving atoms of the burner bounce off the atoms in your finger, and the fast ones slow down a bit; the slower ones bounce back with a little more energy than they started with. Biological systems have evolved to pay attention to this, which is why you can feel it, because too much energy in our atoms is a dangerous thing. Chemical bonds break when they are heated too much; that's what cooking is. Burning your finger by touching a hot electric stove burner is an example of heat **conduction**, the easiest form of heat flux to envision.

Light

A thermos bottle is designed to slow the flow of heat through its walls. You can put warm stuff in there to keep it warm or cool stuff in it to stay cool. Thermos bottles have two walls, one inside and one outside. Between the two walls is an insulator. A **vacuum**, a space with no air in it, is a really good insulator because it has no molecules or atoms of gas in it to carry heat energy between the inner and outer walls of the thermos. Of course, there will still be heat conduction along the walls. But think about a planet. There are no walls connecting a planet with anything. The space between the Earth and the sun is a pretty good vacuum. We know how warm it is in the sunshine, so we know that heat flows from the sun to the Earth. And yet separating the sun from the Earth is 150 million kilometers of vacuum. How can heat travel through this?

Electromagnetic waves carry energy through a vacuum.

The energy travels by means of **light**. Electrons and protons have a property called **electric charge**. What an electric charge is, fundamentally, no one can tell you, but electric charges interact with each other through space via a property of the vacuum called the **electric field**. A positive electric field attracts a negatively charged electron. This is how a TV tube hurls electrons toward the screen in a picture-tube television, in the old days, before electronic flat screens, by releasing an electron and then pushing it around with the electric field.

The electric field interacts with another property of the vacuum called the **magnetic field**. If the strength of the electric field at some location, measurable in volts, changes with time, if the voltage is oscillating up and down, for example, that will cause a change in the magnetic field, such as a compass would point to. This is how an electromagnet works, converting electrical field energy into magnetic field energy. Going the other direction, if the magnetic field changes, it can produce an electrical field. This is how an electrical generator works.

It turns out that the electric and magnetic fields in the vacuum fit together to form a closed, self-replicating cycle like the ringing of a bell or the propagation of a nonbreaking wave in water. An up-and-down oscillation in the electric field will cause a complementary oscillation in the magnetic field, which reinforces the electric field in turn. The disturbances created by the electric and magnetic fields propagates the pattern in the fields, a self-reproducing wave moving through space at a particular velocity, the **speed of light**, written as the letter ***c***. The bundle of electric and magnetic waves can, in principle, hurl through the vacuum forever, carrying energy with it.

A light wave rings like a bell.

The ringing of the electromagnetic field in light can come in a range of different frequencies. **Frequencies**, of oscillators or of light waves, have units of cycles per second and are denoted by the Greek letter ν, pronounced "nu". It turns out that all different frequencies of light travel at the same speed in a vacuum. Within some nonvacuum media, such as air, water, or glass, different frequencies of light might vary in their speeds a little, which is how a prism separates white light into its component colors. But in a vacuum, all light travels at the same speed. The speed of light in a vacuum is a fundamental constant of nature.

The constancy of the speed of light in a vacuum makes it possible to relate the frequency of the light to its **wavelength**, the distance between the crests of a wave. You can figure out what the relationship is between frequency and wavelength by thinking geometrically, imagining the wavy line in Figure 2-1 to be moving past us at some speed, c. If the crests are 1 cm apart and they are moving at 10 cm/second, then 10 crests would move past us every second. Alternatively, you can make use of units. Pay attention to units, and they will lead you to virtue, or at least to the right answer most of the time. Problem: Assemble the two things we know, ν and c, such that the units combine to be the same as the units of λ. Solution:

$$\lambda\left[\frac{\text{cm}}{\text{cycle}}\right] = \frac{c\left[\frac{\text{cm}}{\text{sec}}\right]}{\nu\left[\frac{\text{cycle}}{\text{sec}}\right]}$$

Don't take my word for it. Check and make sure the units are the same on both sides of the equation.

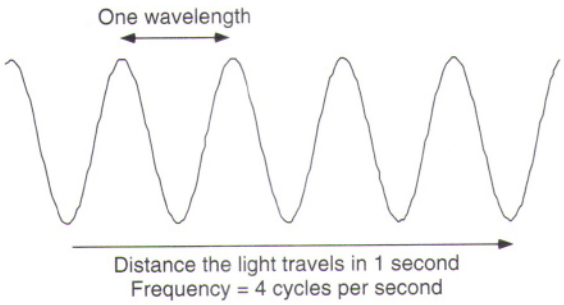

Figure 2-1 The frequency and wavelength of light are related to each other by the speed of light, which in a vacuum is the same for all different types of light, $3.0 \cdot 10^{10}$ cm/s.

Scientists who discuss infrared (IR) light often use a third way of describing different colors, called the **wave number**, defined as the number of cycles per centimeter of length. This is simply a matter of convenience; when IR light is described by its wave number, it will have a nice, easy-to-remember number; for example, the CO_2 absorption of IR light that is responsible for most of its greenhouse effect comes at a wave number value of about 700 cycles/cm. We will use the letter ***n*** to designate the wave number. How can we construct the units of n from the building blocks of λ, ν, and c? By looking at the units, it looks like all we need is λ:

$$n\left[\frac{\text{cycles}}{\text{cm}}\right] = \frac{1}{\lambda\left[\frac{\text{cm}}{\text{cycle}}\right]}$$

Different frequencies of light all have the same fundamental nature; they are all waves of the same essential physics. Figure 2-2 shows the names assigned to different types of light, based on their frequencies. Of course, if you know the frequency of light, you know its wavelength and wave numbers also, so the figure has "mile markers" in these units, too.

Our eyes are sensitive to light in what we pragmatically call the visible range. Light of higher frequencies than this carries enough energy to break chemical bonds, so it can be dangerous to us as chemical creatures. **Ultraviolet light**, or **UV**, causes sunburn, and **x-rays** and **gamma radiation** can do deeper and more extensive chemical damage. At longer wavelengths than the visible, we find the **infrared** range, or **IR**. Objects at room temperature glow with IR light. Heat lamps at the skating rink warm people up by shining invisible IR light on them.

All that energy whizzing around space in the form of coupled electric and magnetic field waves would be of little interest to the energy balance of a planet if the waves did not give up or carry off energy. There are a number of different mechanisms by which light may interact with matter, but IR light interacts mostly with vibrations of the chemical bonds of a molecule, coupled by means of the electric field that they share (Figure 2-3).

Imagine matter as constructed with charged oscillators all over its surface, little charged weights sticking out on little springs that stretch and contract. Each little oscillator will have a frequency with which it will "ring." Incoming energy in the form of light brings with it an electric field oscillating up and down; voltage goes plus, minus, plus, minus. If the frequency of the cycles of the electric field from the light is about the same as the frequency of the oscillations of the chemical bonds, the light can be absorbed. Its energy is transferred into **vibrational energy** of the matter.

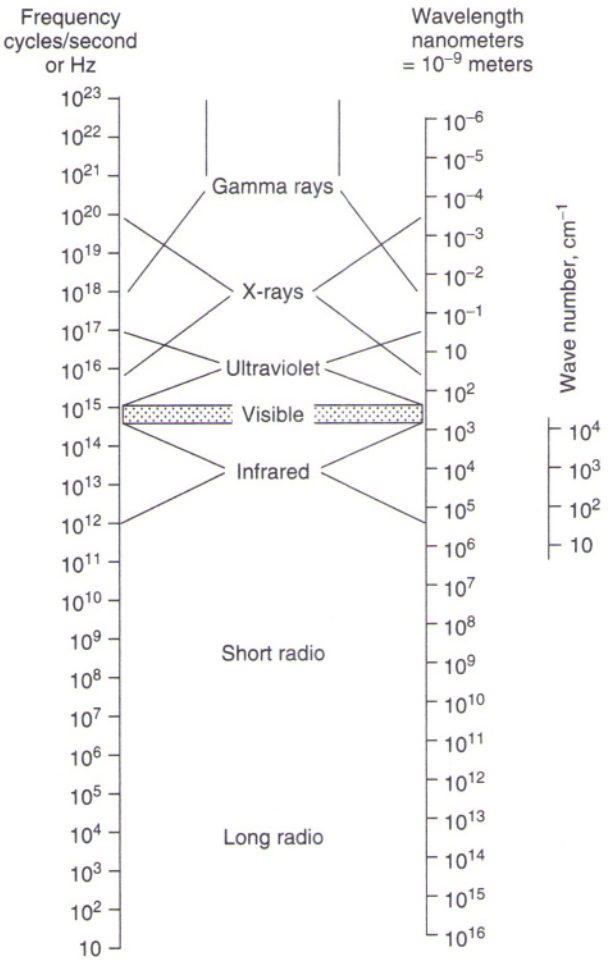

Figure 2-2 The electromagnetic spectrum.

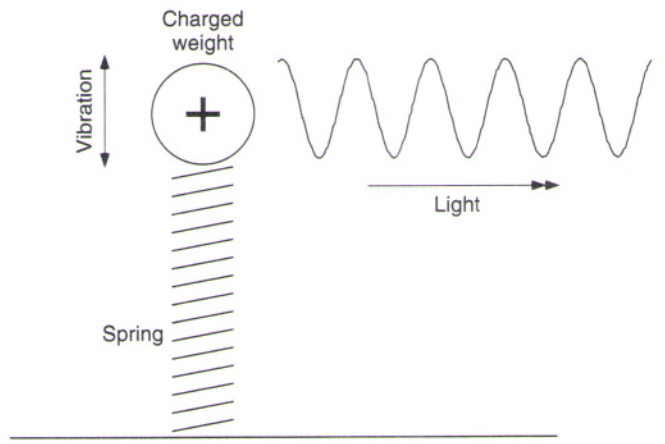

Figure 2-3 A charged oscillator interacting with light.

This mechanism of energy transfer is a two-way street. If energy can flow from the light to the oscillator, it will also be able to go the other way, from the oscillator to light. The vibrational energy of the oscillator is what we have been calling its temperature. Any matter that has a temperature above absolute zero (zero degrees on the Kelvin scale) will have energy in its oscillators that it may be able to use to create light. The two-way street character of this process is important enough that it is given the name **Kirchoff's law**. Try an experiment of singing a single note into a piano with dampers off. When you stop, you will hear the strings echo back the note you sang into it. This sound wave to string vibration energy transfer is a two-way street as well.

Blackbody Radiation

Where can one see electrical energy traveling the other way, from matter into light? One example is a red-hot electric burner, which shines light you can see. The light derives its energy from the vibrations or thermal energy of the matter. We normally do not think of it, but it turns out that your electric burner continues to shine even when the stove is at room temperature. The difference is that the room temperature stove emits light in frequencies that our eyes cannot see, down in the IR.

If a chunk of matter has oscillators that vibrate and can interact with light at all possible frequencies, it is called a **blackbody**. The light that is emitted by a blackbody is called **blackbody radiation**. Most solids and liquids at the surface of the Earth are pretty good blackbodies, but gases in the atmosphere are not blackbodies; they only interact with specific frequencies of light. They are like pianos with most of the strings missing.

> A blackbody is like a musical instrument with all the notes.

Blackbody radiation is made up of a characteristic distribution of frequencies (colors) of infrared light. Figure 2-4 shows a plot with axes of the intensity of light in the y-direction and frequency in the x-direction. The units of intensity look a bit complicated; they are $\frac{\text{Watts}}{\text{m}^2\text{-wave number}}$. The unit on the top of the fraction is **Watts**, the same kind of Watts that describe hairdryers and audio amplifiers. A Watt is a rate of energy flow, defined as Joules per second, where a Joule is an amount of energy, such as might be carried in a battery or a candy bar. The meters squared (m^2) on the bottom of the fraction is the surface area of the object. The unit of wave numbers on the bottom of the fraction allows us to divide the energy up according to the different wave-number bands of light; for instance, all the light between 100 and 101 cm^{-1} carries this many W/m^2 of energy flux, between 101 and 102 cm^{-1} carries that many W/m^2, and so on. The total flux of energy can be calculated by adding up the bits from all the different slices of the light spectrum. The total energy of the wave number slice is I times the width of the slice in wave numbers, resulting in an energy flux in units of W/m^2. You could cut the plot out with a pair of scissors and weigh the inside piece to determine its area, which would then be proportional to the total energy emitted in all frequencies of light.

A plot of intensity versus wavelength of light is called a spectrum. The IR light emission spectrum of a blackbody depends only on the temperature of the object. There are two things you should notice about the shapes of the curves in Figure 2-4. First, as the temperature goes up, the peaks of the curves move to the right, toward visible light. Second, as the temperature of the object goes up, the total energy emitted by the object goes up, which you can see by the fact that the areas under the curves in Figure 2-4 get larger as the temperature rises. There is an equation that tells how quickly energy is radiated from an object. It is called the **Stefan-Boltzmann equation**, and we are going to make extensive use of it. Get to know it now! The equation is:

$$I = \varepsilon \sigma T^4 \tag{2-1}$$

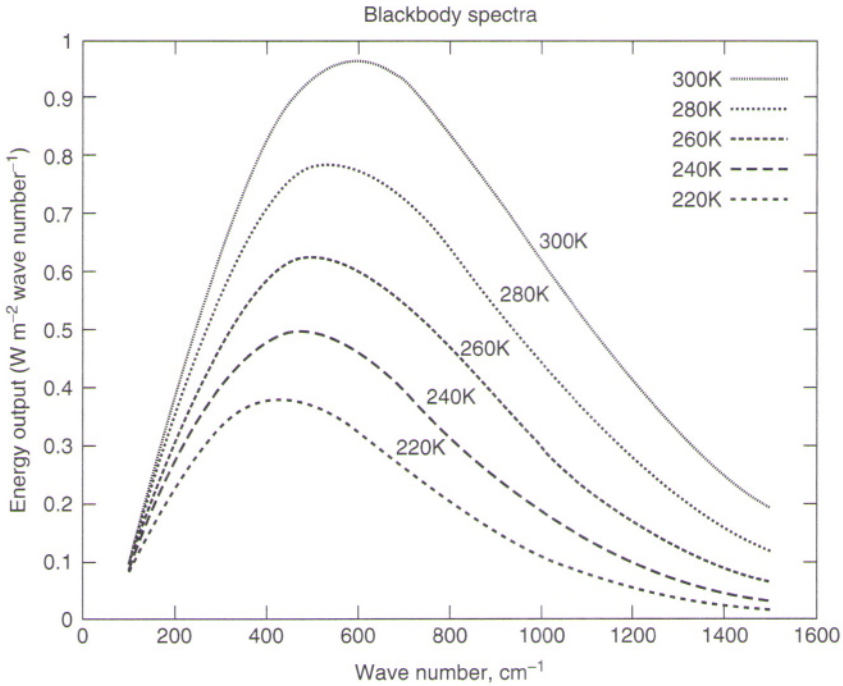

Figure 2-4 The intensity of light emitted from a blackbody as a function of the wave number (cycles per centimeter) of the light. There are several spectra shown for different blackbody objects at different temperatures. A warmer object emits more radiation than a cooler one.

The **intensity** of the light is denoted by I and represents the total rate of energy emission from the object at all frequencies, in units of Watts/m^2. The Greek letter epsilon (ε) is the **emissivity**, a number between zero and one describing how good a blackbody the object is. For a perfect blackbody, $\varepsilon = 1$, and the lower bound is $\varepsilon = 0$. Sigma (σ) is a fundamental constant of physics that never changes, a number you can look up in reference books, called the **Stefan-Boltzmann constant**. T is the temperature in **Kelvins**, and the superscript 4 is an exponent, indicating that we have to raise the temperature to the fourth power. The Kelvin temperature scale begins with 0 K when the atoms are vibrating as little as possible, a temperature called **absolute zero**. There are no negative temperatures on the Kelvin scale.

A hot object emits much more light than a cold object.

One of the many techniques of thinking scientifically is to pay attention to units. Here is Equation 2-1 again, with units of the various terms specified in the square brackets:

$$I\left[\frac{\text{W}}{\text{m}^2}\right] = \varepsilon[\text{unitless}]\ \sigma\left[\frac{\text{W}}{\text{m}^2\ \text{K}^4}\right] T[\text{K}]^4$$

The unit of the intensity I is watts of energy flow per square meter. The meters squared on the bottom of that fraction is the surface area of the object that is radiating. The area of the Earth, for example, is $5.14 \cdot 10^{14}$ m^2. Temperature is in Kelvins, and ε has no units; it is just a number ranging from 0 to 1; 0 for an object that emits nothing and 1 for a perfect blackbody.

Figure 2-5 A photograph taken in infrared light.

Here is the important point: The units on each side of this equation must be the same. On the right-hand side, K^4 cancels leaving only W/m^2 to balance the left-hand side. In general, if you are unsure how to relate one number to another, the first thing to do is to listen to the units. They will guide you to the right answer. We will see many more examples of units in our discussion, and you may rest assured I will not miss an opportunity to point them out.

IR-sensitive cameras allow us to see what the world looks like in IR light. The clear lenses in the glasses of the guys in Figure 2-5 are cooler than their skins and, therefore, darker in the IR. How much darker they will be can be estimated from Equation 1 to be

$$\frac{I_{\text{cheek}}}{I_{\text{glasses}}} = \frac{\varepsilon_{\text{cheek}}\ \sigma\ T_{\text{skin}}^4}{\varepsilon_{\text{glasses}}\ \sigma\ T_{\text{glasses}}^4}$$

The Stefan-Boltzmann constant σ is the same on both top and bottom; σ never changes. The emissivity ε might be different between the skin and the glasses, but let us assume they are the same. This leaves us with the ratio of the brightnesses of the skin and glasses equal to the ratio of temperatures to the fourth power, maybe $(285\,\text{K}/278\,\text{K})^4$, which is about 1.1. The skin shines 10% more brightly than the surface of the coat, and that is what the IR camera sees.

The other thing to notice about the effect of temperature on the blackbody spectra in Figure 2-4 is that the peaks shift to the right as the temperature increases. This is the direction of higher-frequency light. You already knew that a hotter object generates shorter wavelength light because you know about red hot, white hot. Which is hotter? White hot, of course; any kid on the playground knows that. A room temperature object (say 273 K) glows in the IR, where we cannot see it. An electric stove set on High (400–500 K) glows in shorter wavelength light, which begins to creep up into the visible part of the spectrum. The lowest energy part of the visible spectrum is red light. Get the object hotter, say, the temperature of the surface of the sun (5,000 K), and it will fill all wavelengths of the visible part of the spectrum with light. Figure 2-6 compares the spectra of the Earth and the sun. You can see that sunlight is visible while "Earth light" (more usually referred to as **terrestrial radiation**) is IR. Of course, the total energy flux from the sun is much higher than it is from Earth. Repeating the calculation we used for the IR photo, we can calculate that the ratio of the fluxes is $(5{,}000\,\text{K}/273\,\text{K})^4$, or about 10^5. The two spectra in Figure 2-6 have been scaled by dividing each curve by the maximum value that the curve reaches, so that the top of each peak is at a value of one. If we had not done

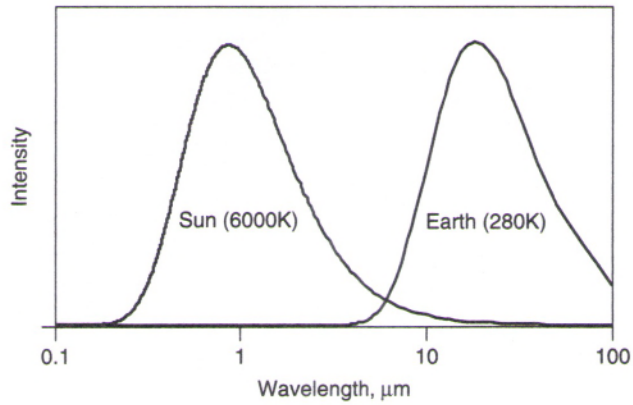

Figure 2-6 The shapes of the blackbody spectra of Earth and the sun. The Earth spectrum has been expanded to reach the same peak intensity as the solar spectrum, so the two can be compared on the same plot. The point is that the sun shines in visible light, whereas the Earth shines in infrared light.

that, the area under the Earth spectrum would be 100,000 times smaller than the area under the sun spectrum, and you would need a microscope to see the Earth spectrum on the figure.

Red hot, white hot.

It is not a coincidence that the sun shines in what we refer to as visible light. Our eyes evolved to be sensitive to visible light. The IR light field would be much more complicated for an organism to measure and understand. For one thing, the eyeball, or whatever light sensor the organism has, will be shining IR light of its own. The organism measures light intensity by measuring how strongly the incoming light deposits energy into oscillators coupled to its nervous system. It must complicate matters if the oscillators are losing energy by radiating light of their own. IR telescopes must be cooled to make accurate IR intensity measurements. Snakes are able to sense IR light. Perhaps this is possible because their body temperatures are colder than those of their intended prey.

TAKE-HOME POINTS

- Light carries energy through the vacuum of space.
- If an object can absorb light, it can also emit light.
- An object that can emit all frequencies of light is called a blackbody, and it emits light energy at a rate equal to $\varepsilon \sigma T^4$.

STUDY QUESTIONS

1. Following the units, find the formula to compute the frequency of light given its wavelength or its wave number from its frequency.
2. Draw a blackbody radiation spectrum for a hot object and a cold object. What two things differ between the two spectra?

3. Use the Stefan-Boltzmann equation to compare the energy fluxes of the hot object and the cold object as a function of their temperatures.
4. What would an emission spectrum look like for an object that is not a blackbody?
5. How does the Stefan-Boltzmann equation deal with an object that is not a blackbody?

FURTHER READING

Blackbody radiation was a clue that something was wrong with classical physics, leading to the development of quantum mechanics. Classical mechanics predicted that an object would radiate an infinite amount of energy, instead of the $\varepsilon \sigma T^4$, as we observe it to be. The failure of classical mechanics is called the ultraviolet catastrophe, and you can read about it, lucidly presented but at a rather high mathematical level, in the *Feynman Lectures on Physics*, Volume 1, Chapter 41. My favorite book about quantum weirdness, the philosophical implications of quantum mechanics, is *In Search of Schrodinger's Cat* by John Gribbon, but there are many others.

Exercises

1. A joule (J) is an amount of energy, and a watt (W) is a rate of using energy, defined as $1\,W = 1\,J/s$. How many Joules of energy are required to run a 100-W light bulb for one day? Burning coal yields about $30 \cdot 10^6$ J of energy per kilogram of coal burned. Assuming that the coal power plant is 30% efficient, how much coal has to be burned to light that light bulb for one day?
2. A gallon of gasoline carries with it about $1.3 \cdot 10^8$ J of energy. Given a price of $3 per gallon, how many Joules can you get for a dollar?

   Electricity goes for about $0.05 per kilowatt hour. A kilowatt hour is just a weird way to write Joules because a watt is a joule per second, and a kilowatt hour is the number of Joules one would get from running 1000 W times one hour (3,600 seconds). In the form of electricity, how many Joules can you get for a dollar?

   A standard cubic foot of natural gas carries with it about $1.1 \cdot 10^6$ J of energy. You can get about $5 \cdot 10^5$ British thermal units (BTUs) of gas for a dollar, and there are about 1,030 BTUs in a standard cubic foot. How many Joules of energy in the form of natural gas can you get for a dollar?

   A ton of coal holds about $3.2 \cdot 10^{10}$ J of energy and costs about $40. How many Joules of energy in the form of coal can you get for a dollar?

   Corn oil costs about $0.10 per fluid ounce wholesale. A fluid ounce carries about 240 dietary calories (which a scientist would call kilocalories). A calorie is about 4.2 J. How many Joules of energy in the form of corn oil can you get for a dollar?

   Rank these as energy sources, cheap to expensive. What is the range in prices?
3. This is one of those job-interview questions to see how creative you are, analogous to one I heard: "How many airplanes are over Chicago at any given time?" You need to make stuff up to get an estimate and demonstrate your management potential. The question is: What is the efficiency of energy production from growing corn?

Assume that sunlight deposits 250 W/m^2 of energy on a corn field, averaging over the day-night cycle. There are 4.186 J per calorie. How many calories of energy are deposited on a square meter of field over the growing season? Now guess how many ears of corn grow per square meter, and guess what the number of calories is that you get for eating an ear of corn. The word *calorie*, when you see it on a food label, actually means *kilocalories* (thousands of calories), so if you guess 100 food-label calories, you are actually guessing 100,000 true calories, or 100 kcal. Compare the sunlight energy with the corn energy to get the efficiency.

4. The Hoover Dam produces $2 \cdot 10^9$ W of electricity. It is composed of $7 \cdot 10^9$ kg of concrete. Concrete requires 1 MJ of energy to produce per kilogram. How much energy did it take to produce the dam? How long is the "energy payback time" for the dam?

   The area of Lake Mead, formed by Hoover Dam, is 247 mi^2. Assuming 250 W/m^2 of sunlight falls on Lake Mead, how much energy could you produce if instead of the lake you installed solar cells that were 12% efficient?

5. It takes approximately $2 \cdot 10^9$ J of energy to manufacture 1 m^2 of crystalline-silicon photovoltaic cell. (Actually, the number quoted was 600 kWhr. Can you figure out how to convert kilowatt hours into Joules?) Assume that the solar cell is 12% efficient, and calculate how long it would take, given 250 W/m^2 of sunlight, for the solar cell to repay the energy it cost for its manufacture.

6. We are supposed to eat about 2,000 dietary calories per day. How many watts is this?

7. Infrared light has a wavelength of about 10 μm. What is its wave number in cm^{-1}?

   Visible light has a wavelength of about 0.5 μm. What is its frequency in Hz (cycles per second)?

   FM radio operates at a frequency of about 40 kHz. What is its wavelength?