Empirical Equations for Some Soil Hydraulic Properties

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The soil moisture characteristic may be modeled as a power curve combined with a short parabolic section near saturation to represent gradual air entry. This two-part function—together with a power function relating soil moisture and hydraulic conductivity—is used to derive a formula for the wetting front suction required by the Green-Ampt equation. Representative parameters for the moisture characteristic, the wetting front suction, and the sorptivity, a parameter in the infiltration equation derived by Philip (1957), are computed by using the desorption data of Holtan et al. (1968). Average values of the parameters, and associated standard deviations, are calculated for 11 soil textural classes. The results of this study indicate that the exponent of the moisture characteristic power curve can be predicted reasonably well from soil texture and that gradual air entry may have a considerable effect on a soil's wetting front suction.

INTRODUCTION

Recent papers by *Amerman* [1973] and *Philip* [1975] have pointed out the importance of including scientific knowledge about soil physics in large-scale hydrologic investigations. For example, to incorporate principles of soil physics into a rainfall-runoff model, one may use either a numerical solution of the unsaturated flow equation or a simple infiltration equation such as that given by *Green and Ampt* [1911] or that derived by *Philip* [1957]. For the first approach the moisture characteristic (the relationship between suction ψ and volumetric water content θ) and the conductivity function (the relationship between the unsaturated hydraulic conductivity K and θ) must be known. For the second approach, composite hydraulic parameters, specifically the Green-Ampt wetting front suction ψ_f and Philip's sorptivity S, can be computed or estimated directly from specified functions of ψ, K, and θ.

The necessity of having to specify relationships among ψ, K, and θ presents a significant problem in hydrology because of the difficulty in obtaining measurements of these parameters and in the representation of the data once they have been collected. A power curve has proved to be a convenient descriptor for the moisture characteristic, and a method presented by *Campbell* [1974] allows the conductivity function to be estimated from this power curve with a matching factor of the saturated hydraulic conductivity K_s. With this approach, four parameters (the fitted coefficient and exponent of the power curve plus θ_s and K_s) give a description of the hydraulic properties of a soil. As will be shown below, this description is improved by introducing an additional parameter to account for gradual air entry near saturation. Knowledge of how these parameters vary with soil properties would be of benefit in hydrologic studies where direct experimental determination of the θ-K-ψ relationships is not feasible and in simulation studies where properties for a 'typical' sand, loam, or clay are needed.

In this paper we show how the power curve representing the two main hydraulic properties is a useful form for hydrologic problems. Specifically, our objectives are (1) to discuss the problem of using the power curve to represent ψ near saturation and to offer an empirical solution, (2) to show how the resulting equations lead to expressions for ψ_f and S, (3) to determine if the suction data reported by *Holtan et al.* [1968] for over 1800 soils conform reasonably well to the power curve, (4) to examine the relationship between the fitted parameters and the soil texture, and (5) to compute representative ψ_f and S values for each textural class.

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EQUATIONS FOR SOIL HYDRAULIC PROPERTIES

The power curve representing the moisture characteristic is

$$\psi = \psi_s W^{-b} \tag{1}$$

with the soil wetness W equal to θ/θ_s, where θ_s is the saturated water content or, in this study, the total porosity. Both ψ_s, the 'saturation' suction, and the exponent b are empirical and must be estimated. *Gardner et al.* [1970] fitted the power curve to suction and diffusivity data and then calculated K as a power curve. *Campbell* [1974] derived a simple formula for k, the relative conductivity ($= K/K_s$). His formula is partly empirical, by using (1), and partly theoretical, by considering pore size distributions within the soil. This equation is $k = W^{2b+2}$, but from the analysis of several soils, best results were obtained where the exponent was increased by 0.7. Consequently, Campbell suggested

$$k = W^{2b+3} \tag{2}$$

as a working relationship. It is important to note that (2) has proved to be reasonably accurate over a wide range of b values (0.17–13.6) and for W values near saturation, in spite of the fact that (1) does not appear to be accurate in this region [*Campbell*, 1974].

The use of (1) implies a sharp discontinuity in suction, or tension, near saturation. While some coarse-grained sands may have a small suction at $W = 1$, most soils, particularly medium- and fine-textured ones, show a gradual air entry region near saturation. Thus we suggest a modification of (1) to account for this gradual air entry. Considering a general moisture characteristic plot, there exists a point where $d\psi/dW$ changes from an increasing to a decreasing function as W decreases. This inflection point is assigned the coordinates (W_i, ψ_i), and the interval $W_i \leq W \leq 1$ can be described by the parabola:

$$\psi = -m(W - n)(W - 1) \tag{3}$$

The parameters m and n are calculated such that (3) passes through points (W_i, ψ_i) and (1, 0) and that $d\psi/dW$ of both (1) and (3) are equal at the inflection point. The expressions for the parameters m and n are

$$m = \frac{\psi_i}{(1 - W_i)^2} - \frac{\psi_i b}{W_i(1 - W_i)}$$

$$n = 2W_i - \left(\frac{\psi_i b}{m W_i}\right) - 1$$

This parabola represents the W versus ψ curve only if $m > 0$, which requires that $W_i > b/(b + 1)$. Normally, this restriction on W_i is not a problem because soils with large b values typically may be represented by an inflection wetness near saturation. With this condition in mind, either W_i or ψ_i may be chosen independently so as to locate the inflection point best with respect to available data. A qualitative illustration of the moisture characteristic appears in Figure 1.

Equations (1) and (3) give the relationship between ψ and W, and (2) provides the K versus W curve if K_s is known. Note that the parameter b can be estimated from suction wetness data, so that unsaturated hydraulic conductivity need not be measured directly. From (1) and (3) the parameters for the Green-Ampt equation and for the Philip equation can be derived.

Neuman [1976] derived the Green-Ampt equation directly from Darcy's law. In so doing, he defined the wetting front suction

$$\psi_f = \int_0^{\psi_{ic}} k\, d\psi \qquad (4)$$

where ψ_{ic} is the initial soil tension. To solve (4), k must be written as a function of ψ. Because the modified power curve for ψ is a two-part function, the k versus ψ curve, illustrated in Figure 2, is obtained from (1) and (2) for $\psi \geq \psi_i$ and from (2) and (3) for $\psi < \psi_i$. Integration of both parts of the curve and addition yield

$$\psi_f = \left(\frac{b}{b+3}\right)(k_i\psi_i - k_{ic}\psi_{ic}) - \left(\frac{2b+3}{2b+5}\right)m(1 - W_i^2 k_i)$$
$$+ \left(\frac{2b+3}{2b+4}\right)m(1+n)(1 - W_i k_i) - mn(1 - k_i) + k_i\psi_i \qquad (5)$$

where k_i and k_{ic} are the relative conductivities corresponding to the suctions ψ_i and ψ_{ic}. For practical purposes, $k_{ic}\psi_{ic}$ may be deleted because k_{ic} is so small for even moderately drained soils. If the parabolic section of the suction curve is neglected, only the first and the last terms on the right-hand side of (5) are retained, and $\psi_i = \psi_s$ and $k_i = 1$. With this assumption,

$$\psi_f = [(2b + 3)/(b + 3)]\psi_s \qquad (6)$$

A first approximation for sorptivity is derived by equating the Green-Ampt equation (considering a surface pressure of zero) to Philip's two-term infiltration equation [*Collis-George*, 1977]:

$$S = [2K_s\psi_f\theta_s(1 - W_{ic})]^{1/2} \qquad (7)$$

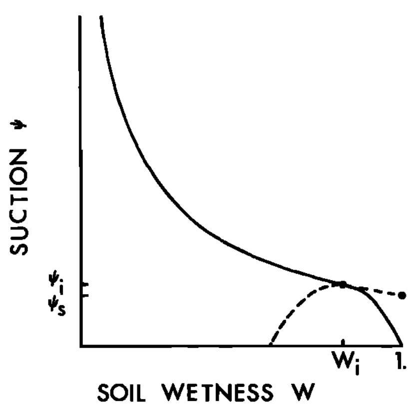

Fig. 1. The moisture characteristic using (1) and (3) for the hyperbolic and parabolic sections, respectively. The broken line segments are disregarded.

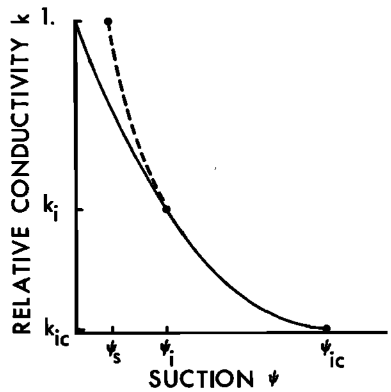

Fig. 2. Relative conductivity versus suction. The area beneath the solid line is the wetting front suction. The broken line segment represents k if gradual air entry is not included in the moisture characteristic.

Brutsaert [1976] compared the solution of (7) with the value resulting from an exact solution of the unsaturated flow equation. The resulting error was 2.9%, which is sufficiently accurate for many purposes.

METHODS

The power curve given by (1) was applied to the desorption data reported by *Holtan et al.* [1968] in order to explore the variability of soil hydraulic parameters. The actual data are not sufficient to estimate ψ_i for (3), so choice of the coordinate (W_i, ψ_i) was made after (1) was fit to the measured data. The sensitivity of other calculated parameters to this choice was investigated to assess the importance of accurate data at low suctions.

The soil samples used to generate these data were collected from 34 localities throughout the United States. In each testing area a variety of sampling sites was chosen, and all horizons were subsampled. Moisture retention was measured at tensions of 0.1, 0.3, 0.6, 3, and 15 bars on a weight-weight basis. For each soil the W values were calculated by using the bulk density obtained by measuring the displacement of the sample dried to a 3-bar tension. All tensions were converted to centimeters of water.

For each soil, ψ_s and b were determined by taking the logarithm of both sides of (1) and performing a linear regression. The residual error at each tension level was calculated as the difference between the experimentally controlled suction and the calculated suction. The log transformations weight the observations such that the sums of the squared relative errors (percent errors) are minimized. For the sake of comparison, a nonlinear parameter estimation routine was also tried. Typically, the nonlinear technique resulted in a relatively close fit at high tensions but an unacceptably poor fit at the 0.1-bar tension.

Not all soils were included in the final statistics. The results for rocky soils (predominantly C horizons) were deleted because they were too erratic. Any soil with a b value greater than 25 was deleted because we felt that exponents of this magnitude were anomalous. For numerous soils the calculated W exceeded unity at 0.1-bar tension. These soils, too, were deleted. Consequently, 1446 soils were analyzed out of an initial set of over 1800.

Representative ψ_f and S values for each soil texture were calculated by setting b equal to its mean value $\bar{b}$ and ψ_s equal to the antilog of the mean log ψ_s, designated as $\bar{\psi}_s(\log)$. This particular average value was chosen for reasons described below. Nothing in the data set indicated where the tension inflection point might occur. For operational purposes the inflection point may be considered equal to the air entry point. *Rogowski* [1971] stated that this point usually occurs in the interval $0.8 < W < 1$ and that $W = 0.9$ is a useful estimate. Because of the restriction on W_i imposed by the exponent b, W_i was maintained at 0.92.

The sorptivity calculations used the average K_s values for each textural class reported by *Li et al.* [1976]. The initial moisture deficit for each sorptivity was computed as the difference between the average total porosity and the moisture representing 500-cm initial suction.

A somewhat arbitrary procedure was developed to compare the parameters with soil texture. Although *Holtan et al.* [1968] did assign a textural class to each soil, no actual particle size distribution data were provided. In order to have some index of texture, we used the relative amount of clay for each soil class because it is generally acknowledged that clay greatly affects a soil's hydraulic properties. Using the *U.S. Department of Agriculture* [1951, p. 209] textural triangle, we located the center of each textural class and read the corresponding clay fraction. For purposes of comparison, soils are ordered according to this mean clay fraction.

RESULTS AND DISCUSSION

The overall accuracy of (1) in fitting the data is shown by the mean residuals in Table 1. The negative mean residual at 3060-cm suction indicates that on the average, the power curve underestimates ψ at this level. The positive mean residuals at the other suctions indicate overestimation of ψ for a majority of the soils. These mean errors appear sizable, but because no replications were performed, it is impossible to assess the effect of experimental errors on the residuals. The fact that the original data set contains numerous identifiable inconsistencies indicates that experimental errors may be a large factor in the computed residuals. If an empirical model is an accurate predictor of a dependent variable, the expected value of the residual is zero. The mean residuals of Table 1 are not significantly different from zero at the 5% level, assuming that the residuals are normally distributed.

The increase in the standard deviations of the residuals with increasing tension is a consequence of the weighting effect of the log transformations. This weighting is actually desirable because for many hydrologic problems the low end of the suction range is most important. Gravity drainage occurs primarily between tensions of zero and 500 cm, and normal plant activity continues at moisture levels up to several thousand centimeters. In contrast, phenomena at the dry end of the moisture characteristic are difficult to model precisely, and because the hydraulic conductivity is reduced by many orders of magnitude, even a rough approximation in this range may be sufficient. Thus regression using log-transformed data is superior to a nonlinear least squares fit in most applications.

The statistics of the soil moisture parameters grouped according to texture appear in Table 2. Moving from coarse to fine soils, $\bar{b}$ increases rather consistently. The one soil fitted with the power curve by *Gardner et al.* [1970] and three of the four examined by *Campbell* [1974] had exponents conforming to these results. The one exception within Campbell's results was Botany sand ($b = 0.17$), which is not a natural soil. The lowest b values obtained from the Holtan data set are about 2. The $\bar{b}$ values are highly correlated with mean clay fraction ($r = 0.98$), but a portion of this correlation coefficient results from regressing average values. In other words, had grain size data

TABLE 1. Average Errors From the Power Curve Moisture Characteristic (Based on Data for 1446 Soils)

Controlled Tension, cm	Mean Residual Tension, cm	Standard Deviation, cm
102	36	63
306	63	125
612	172	360
3,060	−1000	1083
15,300	1404	5172

been provided for each soil and had these data been correlated with individual b values, the regression coefficient would undoubtedly not be as close to unity. Nevertheless, the b exponent is strongly dependent on texture, and texture can be accepted as an indicator of b.

The interpretation of the ψ_s coefficients is less clear. For each soil group the ψ_s distributions have large standard deviations and are strongly positively skewed. For example, a majority of the ψ_s values for sandy loam are centered about 7 cm, but many values range to 50 cm, and some are even greater than 100 cm. Consequently, the 21.8-cm mean is considerably higher than the mode and the median values. The most probable ψ_s is well represented by the $\bar{\psi}_s$(log) average. For the sandy loam class this value is 7.18 cm, which is a good representation of the ψ_s of an 'average' sandy loam. Alternatively, it can be shown that if the W values for each tension are averaged for all the sandy loams and then log-transformed and fitted, the resulting ψ_s is close to $\bar{\psi}_s$(log). Although both $\bar{\psi}_s$ and $\bar{\psi}_s$(log) increase with finer soils, neither is well correlated with the mean clay fraction; hence texture is not a good indicator of ψ_s.

The ψ_f values are calculated from $\bar{\psi}_s$(log). To assess the relative importance of ψ_s and the exponent b in the calculation of ψ_f consider (6), which approximates ψ_s without the gradual air entry near saturation. When the range of $\bar{b}$ values is used, the $(2b + 3)/(b + 3)$ coefficient varies from 1.57 to 1.79 in comparison to $\bar{\psi}_s$(log), which ranges from 1.8 to 56.6 cm. Consequently, the representative ψ_f values in Table 2 follow the pattern of $\bar{\psi}_s$(log); however, these wetting front suctions do incorporate an inflection point in the W versus ψ curve, so that ψ_f is affected by W_i too. Examining the average parameters for the typical sand, if W_i is set equal to 1, then ψ_f increases by 17% over the given value. For the clay soil, setting $W = 1$ increases ψ_f by 37%. Decreasing W_i from 0.92 to 0.84 for the sand decreases ψ_f by 14%. For the clay, W_i may not be set to this low value because of the stipulation described earlier. These results demonstrate that changes in the representation of the moisture characteristic near saturation cause considerable changes in ψ_f approximations.

Li et al. [1976] estimated ψ_f for different soil textures by graphing generalized ψ, k plots for each texture and by subsequently integrating the plots graphically. Roughly half of the ψ_f values in Table 2 are similar to those reported by *Li et al.* [1976]. The low ψ_f values tend to agree, while discrepancies occur between the larger ones.

No compensations are made here for hysteresis, although inaccuracies may be introduced because desorption data are used to calculate ψ_f, a parameter used to describe an imbibition process. *Mein and Larson* [1973] divided each desorption suction by 1.6 to represent the imbibition suction. Alternatively, *Brakensiek* [1977] divided the bubbling pressure by 2 to represent the air exit pressure. If these methods were applied to the equations presented here, they would effectively decrease

TABLE 2. Representative Values for Hydraulic Parameters (Standard Deviations in Parentheses)

Soil Texture	Soils	Mean Clay Fraction	$\bar{b}$	$\bar{\psi}_s$, cm	$\bar{\psi}_s$(log), cm	ψ_f, cm	$\bar{\theta}_s$, cm^3/cm^3	K_s,* cm/min	S, cm/min$^{1/2}$
Sand	13	0.03	4.05 (1.78)	12.1 (14.3)	3.50	4.66	0.395 (0.056)	1.056	1.52
Loamy sand	30	0.06	4.38 (1.47)	9.0 (12.4)	1.78	2.38	0.410 (0.068)	0.938	1.04
Sandy loam	204	0.09	4.90 (1.75)	21.8 (31.0)	7.18	9.52	0.435 (0.086)	0.208	1.03
Silt loam	384	0.14	5.30 (1.96)	78.6 (51.2)	56.6	75.3	0.485 (0.059)	0.0432	1.26
Loam	125	0.19	5.39 (1.87)	47.8 (51.2)	14.6	20.0	0.451 (0.078)	0.0417	0.693
Sandy clay loam	80	0.28	7.12 (2.43)	29.9 (37.8)	8.63	11.7	0.420 (0.059)	0.0378	0.488
Silty clay loam	147	0.34	7.75 (2.77)	35.6 (37.8)	14.6	19.7	0.477 (0.057)	0.0102	0.310
Clay loam	262	0.34	8.52 (3.44)	63.0 (51.0)	36.1	48.1	0.476 (0.053)	0.0147	0.537
Sandy clay	19	0.43	10.4 (1.64)	15.3 (17.3)	6.16	8.18	0.426 (0.057)	0.0130	0.223
Silty clay	441	0.49	10.4 (4.45)	49.0 (62.1)	17.4	23.0	0.492 (0.064)	0.0062	0.242
Clay	140	0.63	11.4 (3.70)	40.5 (39.7)	18.6	24.3	0.482 (0.050)	0.0077	0.268

*From *Li et al.* [1976].

ψ_s by a factor of 1.6 or 2, respectively. The suction ψ_f would also be reduced by roughly the same factors. Neither modification was included in calculated values reported here because it is not clear that these procedures apply to the wide range of soils investigated here.

The sorptivity values in Table 2 must be considered cautiously. *Li et al.* [1976] mentioned that their average K_s values are considerably higher than other averages previously reported. Even if they are in some way representative, they may not correspond to the average soils typified by $\bar{b}$ and $\bar{\psi}_s$(log). In addition, S represents only one initial moisture deficit determined by a single initial suction.

Equations (1) and (3) for the moisture characteristic, plus Campbell's equation (2) for hydraulic conductivity, have been useful to us in several different soil moisture models. The procedure for calculating ψ_f presented here provides for parameter estimates for infiltration problems using only limited soil data. The large standard deviations of ψ_s within each textural class indicate that blind use of these average values may give erroneous results. The average values presented here have not been verified and should be used with this limitation in mind. However, it has been shown that the exponent b is statistically related to soil texture, that there is substantial variability in ψ_s within and among textural classes, and that gradual air entry—represented by the inflection point on the W versus ψ curve—may have considerable effect on the calculated wetting front suction.

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