14 Turbulent fluxes

14.1 Chapter summary

The fluxes of sensible and latent heat occur because turbulent mixing of air transports heat and moisture, typically away from the surface. These fluxes are described in terms of transport by mean motion (typically a small term) and by turbulence (the dominant term). Turbulent transport is quantified by the covariance of temperature and moisture fluctuations with vertical velocity fluctuations. Turbulent fluxes of sensible heat, latent heat, and momentum are related to logarithmic profiles of temperature, humidity, and wind near the surface. Monin–Obukhov similarity theory describes profiles and fluxes in the surface layer of the atmosphere and is used to derive the aerodynamic resistances that regulate turbulent fluxes. These resistances and the profiles of temperature, humidity, and wind in the surface layer depend in part on roughness length and displacement height, which vary greatly among land cover types.

14.2 Turbulence

The exchanges of sensible and latent heat between land and atmosphere occur because of turbulent mixing of air and resultant heat and moisture transport. The flow of air can be represented as discrete parcels of air moving vertically and horizontally. These parcels have properties such as temperature, humidity, and momentum (mass times velocity). As the parcels of air move, they carry with them their heat, moisture, and momentum. Turbulence creates eddies that mix air from above downward and from below upward and transports heat and water vapor in relation to the temperature and moisture of the parcels of air being mixed (Fig. 14.1). For example, air near the surface is generally warmer and moister than air above. Downward movements of air, therefore, tend to decrease temperature and humidity. Conversely, upward movements of air carry heat and moisture from the surface higher into the atmosphere.

The importance of turbulence in mixing air in the atmospheric boundary layer near the surface is illustrated with two simple examples. Over the course of a summer day, productive vegetation can absorb 5–6 $g\,C\,m^{-2}$ or more during photosynthesis. This is enough to deplete all the CO_2 in the air to a height of 30 m or so. (The molecular weight of the atmosphere is 28.97 $g\,mol^{-1}$, while that of carbon is 12 $g\,mol^{-1}$. With a CO_2 concentration of 379 parts per million by volume and a density of 1.15 $kg\,m^{-3}$, a cubic meter of air contains about 379×10^{-6} (12/28.97) 1150 $g\,m^{-3} = 0.18\,g\,C\,m^{-3}$. A net uptake of 6 $g\,C\,m^{-2}$ per day would deplete carbon to a height of 33 m.) In practice, however, CO_2 is not depleted because wind mixes the air and replenishes the absorbed CO_2. Likewise, productive vegetation can transpire 5–10 $kg\,m^{-2}$ (5–10 mm) of water on a hot summer day. This would increase the moisture in the first 100 m of the atmosphere by 50–100 $g\,m^{-3}$ if it was not transported away. This is much more water than the atmosphere can hold when saturated. Instead, wind replaces the moist air with drier air.

Near the surface, turbulence generates vertical motions. Turbulence can be generated mechanically due to surface friction (often called forced convection) when wind flows over Earth's surface (Fig. 14.1). The ground, trees, grasses, and other objects protruding into the atmosphere exert a retarding force on the fluid motion of air. The frictional drag imparted on air as it encounters these rough surface elements slows the flow of air near the ground. The reduction in wind speed transfers momentum from the atmosphere to the surface, creating turbulence that mixes the air and transports heat and water from the surface into the lower atmosphere. With greater height above the surface, eddies are larger so that transport of momentum, heat, and moisture is more efficient with height above the surface.

Vertical turbulent motion can also occur due to surface heating and buoyancy (often called free convection). In daytime, the surface is typically warmer than the

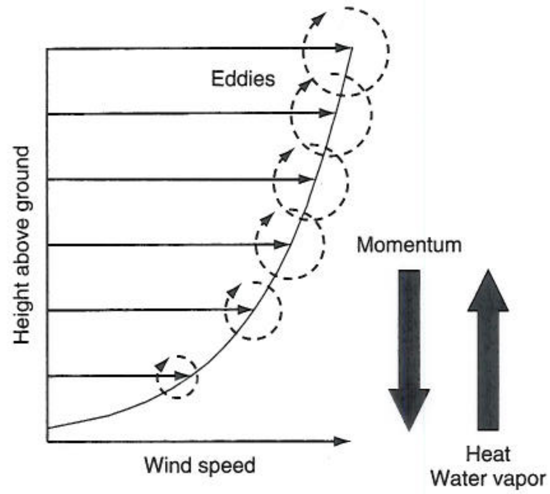

FIGURE 14.1. Conceptual diagram of momentum, heat, and water vapor fluxes. Eddies transfer momentum to the surface, while heat and water are generally carried from the warm, moist surface to the cooler, drier atmosphere.

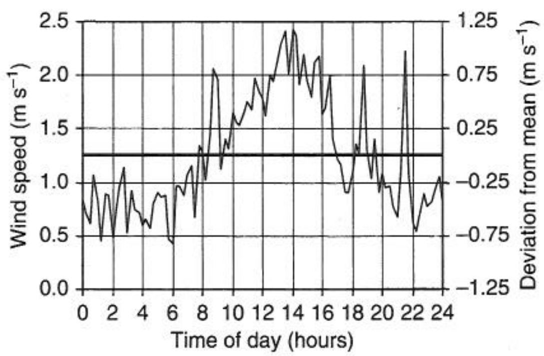

FIGURE 14.2. Wind speed (left-hand axis) and deviation from mean (right-hand axis) measured over the course of a day. The thick solid line shows the diurnal mean.

atmosphere and strong solar heating of the land provides a source of buoyant energy. Warm air is less dense than cold air, and rising air enhances mixing and the transport of heat and moisture away from the surface. In this unstable atmosphere, sensible heat flux is positive, temperature decreases with height, and vertical transport increases with strong surface heating. At night, longwave emission generally cools the surface more rapidly than the air above. The lowest levels of the atmosphere become stable, with cold, dense air trapped near the surface and warmer air above. Sensible heat flux is negative (i.e., toward the surface). Under these conditions, vertical motions are suppressed, and transport is reduced.

14.3 The statistics of turbulence

Turbulent motion in the atmosphere is seen in the patterns of smoke emitted from a chimney, the swirling of dust or other debris, the ripples of waves across a lake, or the fluttering of leaves and flags. Yet while turbulence is easily observed, it is difficult to describe mathematically. A deterministic description of turbulence is difficult because of its chaotic behavior. Instead, the randomness and unpredictability of turbulent motion necessitates the use of statistics to characterize turbulence.

Turbulent flow can be described mathematically by representing an instantaneous measurement as its mean and its fluctuating component. Consider, for example, a quantity a. Its mean over a specified time interval is $\bar{a}$. At any given time during that period, its fluctuation from the mean is $a' = a - \bar{a}$. In other words, a variable can be represented as the sum of its mean and fluctuating components:

$$a = \bar{a} + a' \tag{14.1}$$

The mean represents the component of a that varies slowly over time. The fluctuation from the mean is the turbulent component. The mean of a' is by definition zero. This is evident from (14.1), which shows that the mean of a is

$$\bar{a} = \overline{(\bar{a} + a')} = \overline{(\bar{a})} + \overline{a'} = \bar{a} + \overline{a'} \tag{14.2}$$

Equation (14.2) is true only if $\overline{a'} = 0$. The variance of a is

$$\sigma^2 = \frac{1}{N}\Sigma(a_i - \bar{a})^2 = \overline{a'a'} \tag{14.3}$$

As an example, consider the horizontal wind speed data shown in Fig. 14.2 measured every 15 minutes over the course of a day. Wind speed ranged from a minimum of 0.43 m s^{-1} to a maximum of 2.42 m s^{-1} and averaged 1.25 m s^{-1}. At any given time, the instantaneous wind speed can be characterized by its deviation from the mean.

A parcel of air has scalar properties such as temperature, humidity, or CO_2 concentration that are carried with the parcel as it is mixed horizontally and vertically in the atmosphere. Indeed, the ability to efficiently mix the properties of air is an important characteristic of turbulence. Consequently, quantities co-vary over time. Consider a second quantity $b = \bar{b} + b'$. The average product $\overline{ab}$ is

$$\overline{ab} = \bar{a}\bar{b} + \overline{a'b'} \tag{14.4}$$

The term $\overline{a'b'}$ is also the covariance between the two quantities. That is,

$$\text{cov}(a,b) = \frac{1}{N}\Sigma(a_i - \bar{a})(b_i - \bar{b}) = \overline{a'b'} \tag{14.5}$$

Variables of interest in the atmosphere include: u, the velocity component in the x-direction (termed zonal wind); v, the velocity component in the y-direction (meridional wind); w, the vertical velocity; θ, the potential temperature; and q, the specific humidity. Each of these can be represented as the sum of mean and turbulent components:

$$\begin{aligned} u &= \bar{u} + u' \\ v &= \bar{v} + v' \\ w &= \bar{w} + w' \\ \theta &= \bar{\theta} + \theta' \\ q &= \bar{q} + q' \end{aligned} \quad (14.6)$$

Of particular importance are the covariances $\overline{u'w'}$, $\overline{v'w'}$, $\overline{\theta' w'}$, and $\overline{q'w'}$, which are related to the turbulent fluxes of momentum, heat, and moisture. (Note: potential temperature θ is the temperature a parcel of air would have if it was brought adiabatically from its actual pressure P to a reference pressure P_0 = 1000 hPa, i.e., $\theta = T(P_0/P)^{R_d/C_p}$, where R_d=287 J kg^{-1} K^{-1} is the gas constant for dry air and C_p = 1004 J kg^{-1} K^{-1} is the specific heat of dry air at constant pressure. Potential temperature is used instead of temperature in boundary layer meteorology because it is a quantity that is conserved in vertical motion.)

14.4 Eddy covariance

The vertical flux of a scalar with concentration c at any instant is the product of its concentration times the vertical velocity (i.e., cw) so that the mean flux over a period of time is

$$\overline{cw} = \bar{c}\,\bar{w} + \overline{c'w'} \quad (14.7)$$

That is, the total scalar flux is the sum of transport by mean motion and transport by turbulence. Mean transport is generally small. Instead, the latter term, which is called the turbulent flux, is the dominant term.

Scalars of interest to calculate sensible and latent heat fluxes are potential temperature (θ, K) and specific humidity (q, kg kg^{-1}). Sensible heat flux (H, W m^{-2}) and water vapor flux (E, kg m^{-2} s^{-1}) are

$$\begin{aligned} H &= \rho C_p \overline{\theta' w'} \\ E &= \rho \overline{q'w'} \end{aligned} \quad (14.8)$$

where ρ is the density of air (kg m^{-3}), C_p is the heat capacity of air (J kg^{-1} K^{-1}), and w' has units m s^{-1}. The momentum flux (τ, kg m^{-1} s^{-2}) has zonal and meridional components related to u and v (m s^{-1}):

$$\begin{aligned} \tau_x &= -\rho \overline{u'w'} \\ \tau_y &= -\rho \overline{v'w'} \end{aligned} \quad (14.9)$$

One means to estimate these fluxes, known as eddy covariance, is to measure fluctuations of temperature (θ'), water vapor (q'), wind (u', v'), and vertical velocity (w') and determine the covariance of the variable of interest with w'. Similar methodology is used to measure CO_2 flux or fluxes of other scalars. Eddy covariance has become a standard technique to measure turbulent fluxes at the surface (Baldocchi *et al.* 1988; Aubinet *et al.* 2000; Baldocchi 2003).

14.5 Logarithmic wind profiles

Under near-neutral atmospheric conditions, the wind profile in the atmosphere over flat homogeneous terrain increases logarithmically with height. For example, Fig. 14.3 shows wind speeds measured to a height of 16 m above sparse grassland in southeastern Australia. Wind speed is lowest near the surface and increases with greater height. The logarithmic wind profile is related to the momentum flux, which is nearly constant with height in the layer of the atmosphere near the surface (the lowest 50 m or so).

The logarithmic wind profile can be derived by defining a velocity scale (u_*, m s^{-1}), also called the friction velocity, that is related to surface stress as

$$u_*^2 = \tau/\rho = \left[\left(\overline{u'w'}\right)^2 + \left(\overline{v'w'}\right)^2\right]^{1/2} \quad (14.10)$$

For convenience, the x-axis is oriented along the direction of surface wind so that v = 0. Then,

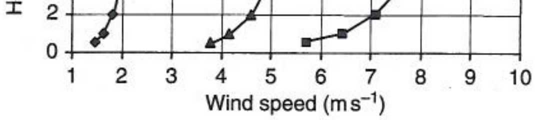

FIGURE 14.3. Wind profile above sparse grassland in southeastern Australia under near-neutral conditions measured at 1630 hours on days 12, 14, and 35 of the Wangara experiment. Data from Clarke *et al.* (1971).

TABLE 14.1. Roughness length of various surfaces

Surface	Roughness length (m)
Soil	0.001–0.01
Grass	
Short	0.003–0.01
Tall	0.04–0.10
Crop	0.04–0.20
Forest	1.0–6.0
Suburban	
Low density	0.4–1.2
High density	0.8–1.8
Urban	
Short building	1.5–2.5
Tall building	2.5–10

Source. Data from Oke (1987, p. 57, p. 298).

$$u_*^2 = \tau/\rho = -\overline{u'w'} \tag{14.11}$$

The momentum flux can be related to the mean vertical gradient of wind as

$$\tau = \rho k u_* z(\partial\bar{u}/\partial z) \tag{14.12}$$

Here, z is height above the surface and $k=0.4$ is the von Karman constant. Combining (14.11) and (14.12) gives

$$(z/u_*)\partial\bar{u}/\partial z = 1/k \tag{14.13}$$

This equation states that when scaled by the velocity scale (u_*) the mean vertical wind gradient ($\partial\bar{u}/\partial z$) depends only on height above the surface (z). Stated another way, the mean vertical wind gradient depends solely on a characteristic velocity scale (u_*) and a characteristic length scale (z).

Integrating $\partial\bar{u}/\partial z$ between two arbitrary heights ($z_2 > z_1$) gives

$$\bar{u}_2 - \bar{u}_1 = (u_*/k)\ln(z_2/z_1) \tag{14.14}$$

The surface is defined as $\bar{u}_1 = 0$ at $z_1 = z_{0M}$ so that wind speed at height z is

$$\bar{u}(z) = (u_*/k)\ln(z/z_{0M}) \tag{14.15}$$

The term z_{0M} is known as the roughness length for momentum. It is the theoretical height at which wind speed is zero. Typical roughness lengths are shown in Table 14.1. Roughness length is generally less than 0.01 m for soil and increases with vegetation.

The height (z) in (14.15) is height above the ground surface. Over some surfaces, however, the protrusion of roughness elements above the surface displaces turbulent flow upward. Trees, for example, extend into the atmosphere from the ground. In this case, height must be adjusted relative to a reference height known as the zero-plane displacement height (d) and (14.15) becomes

$$\bar{u}(z) = (u_*/k)\ln[(z-d)/z_{0M}] \tag{14.16}$$

The displacement height is the vertical displacement caused by surface elements. This displacement height is zero for bare ground and greater than zero for vegetation. The height $z_{0M} + d$ is the height where the wind profile extrapolates to zero and is known as the apparent sink of momentum. The logarithmic wind profile is valid only for heights $z >> z_{0M} + d$ and does not describe wind within plant canopies. Also, it only applies to an extensive uniform surface of 1 km^2 or more.

Vegetation increases surface roughness in relation to canopy height and density. Short grass is a smoother surface than tall grass. Forests are aerodynamically rougher than crops or grasses. It is often assumed that roughness length for vegetation is one-tenth canopy height and displacement height is seven-tenths canopy height (Grace 1983, p. 44; Monteith and Unsworth 1990, p. 117; Campbell and Norman 1998, p. 71).

14.6 Monin–Obukhov similarity theory

The preceding flux–profile relationships can be generalized from near-neutral conditions to all atmospheric conditions and extended to include temperature and humidity profiles. The turbulent fluxes of momentum, sensible heat, and latent heat are nearly constant with height in the layer of the atmosphere near the surface. This region of the atmosphere is known as the surface, or constant flux, layer and generally extends 50 m or so above the ground. The Monin–Obukhov similarity theory relates turbulent fluxes of momentum, sensible heat, and moisture in the surface layer to mean vertical gradients of wind, potential temperature, and water vapor (Brutsaert 1982; Garratt 1992; Arya 2001).

The Monin–Obukhov similarity theory states that when scaled appropriately the dimensionless mean vertical gradients of wind ($\bar{u}$), potential temperature ($\bar{\theta}$), and specific humidity ($\bar{q}$) are unique functions of a buoyancy parameter (ζ):

$$\begin{aligned}
\left[\frac{k(z-d)}{u_*}\right]\frac{\partial\bar{u}}{\partial z} &= \phi_m(\zeta) \\
\left[\frac{k(z-d)}{\theta_*}\right]\frac{\partial\bar{\theta}}{\partial z} &= \phi_h(\zeta) \\
\left[\frac{k(z-d)}{q_*}\right]\frac{\partial\bar{q}}{\partial z} &= \phi_w(\zeta)
\end{aligned} \tag{14.17}$$

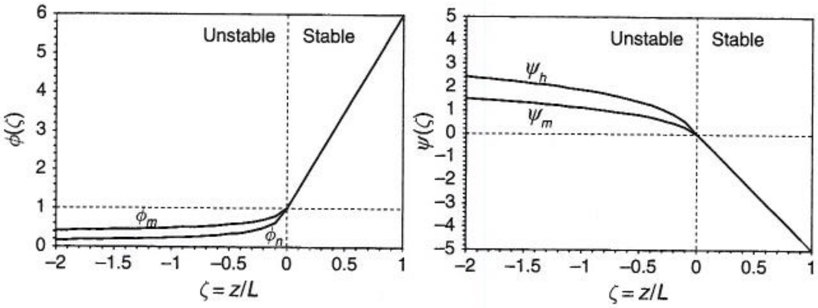

FIGURE 14.4. Similarity functions $\phi_m(\zeta)$ and $\phi_h(\zeta)$ (left) and $\psi_m(\zeta)$ and $\psi_h(\zeta)$ (right) in relation to $\zeta = z/L$.

The functions $\phi_m(\zeta)$, $\phi_h(\zeta)$, and $\phi_w(\zeta)$ are universal similarity functions that relate the constant fluxes of momentum, sensible heat, and water vapor to the mean vertical gradients of wind, temperature, and moisture in the surface layer. The characteristic velocity (u_*), temperature (θ_*), and moisture (q_*) scales are

$$\begin{aligned} u_* u_* &= -\overline{u'w'} = \tau/\rho \\ \theta_* u_* &= -\overline{\theta' w'} = -H/(\rho C_p) \\ q_* u_* &= -\overline{q' w'} = -E/\rho \end{aligned} \quad (14.18)$$

(with the x-axis oriented along the direction of surface wind so that $v = 0$). Equations (14.17) and (14.18) are extensions of (14.11) and (14.13) to account for buoyancy, with similar flux–profile equations for temperature and humidity.

The effect of buoyancy on turbulence is represented by the similarity functions $\phi_m(\zeta)$, $\phi_h(\zeta)$, and $\phi_w(\zeta)$, where

$$\zeta = (z - d)/L \quad (14.19)$$

and L (m) is the Obukhov length scale. The parameter L is given by

$$L = -u_*^3 / \left[k (g/\theta)(H/\rho C_p)\right] \quad (14.20)$$

where $g = 9.81$ m s^{-2} is gravitational acceleration and θ is the potential temperature at the surface. Positive values indicate a stable atmosphere. Negative values indicate an unstable atmosphere. Moist air is less dense than dry air. To account for the effect of water vapor on buoyancy, potential temperature (θ) and sensible heat flux (H) are replaced by virtual potential temperature (θ_v) and virtual sensible heat (H_v), approximated as

$$\begin{aligned} \theta_v &= \theta(1 + 0.61q) \\ H_v &= H + 0.61 C_p \theta E \end{aligned} \quad (14.21)$$

Representative similarity functions are

$$\begin{aligned} \phi_m^2(\zeta) = \phi_h(\zeta) = \phi_w(\zeta) &= (1 - 16\zeta)^{-1/2} && \text{for } \zeta < 0 \text{ (unstable)} \\ \phi_m(\zeta) = \phi_h(\zeta) = \phi_w(\zeta) &= 1 + 5\zeta && \text{for } \zeta \geq 0 \text{ (stable)} \end{aligned} \quad (14.22)$$

These functions have values of <1 when the atmosphere is unstable and >1 when the atmosphere is stable (Fig. 14.4).

Profile equations are obtained by integrating $\partial\bar{u}/\partial z$ (14.17) between two heights ($z_2 > z_1$):

$$\bar{u}_2 - \bar{u}_1 = \frac{u_*}{k}\left[\ln\left(\frac{z_2 - d}{z_1 - d}\right) - \psi_m\left(\frac{z_2 - d}{L}\right) + \psi_m\left(\frac{z_1 - d}{L}\right)\right] \quad (14.23)$$

The surface is defined as $\bar{u}_1 = 0$ at $z_1 = z_{0M} + d$ and $\psi_m[(z_1 - d)/L] = 0$ so that wind speed at height z is

$$\bar{u}(z) = \frac{u_*}{k}\left[\ln\left(\frac{z - d}{z_{0M}}\right) - \psi_m(\zeta)\right] \quad (14.24)$$

Equation (14.24) is analogous to (14.16), but with the function $\psi_m(\zeta)$ to account for atmospheric stability. Similarly, $\partial\bar{\theta}/\partial z$ and $\partial\bar{q}/\partial z$ (14.17) are integrated between height z and the surface:

$$\begin{aligned} \bar{\theta}(z) - \bar{\theta}_s &= \frac{\theta_*}{k}\left[\ln\left(\frac{z - d}{z_{0H}}\right) - \psi_h(\zeta)\right] \\ \bar{q}(z) - \bar{q}_s &= \frac{q_*}{k}\left[\ln\left(\frac{z - d}{z_{0W}}\right) - \psi_w(\zeta)\right] \end{aligned} \quad (14.25)$$

For temperature and specific humidity, surface values are defined as $\bar{\theta}_s$ at $z = z_{0H} + d$ and $\bar{q}_s$ at $z = z_{0W} + d$, where z_{0H} and z_{0W} are the roughness lengths for heat and moisture, respectively. Similar to momentum, these are the effective height at which heat and moisture are exchanged with the atmosphere and are known as the apparent sources of heat and moisture.

Equations (14.24) and (14.25) show that the mean wind, potential temperature, and specific humidity, when scaled appropriately (i.e., by u_*, θ_*, and q_*, respectively), depend on height (z) and length scale (L). From the definitions of u_*, θ_* and q_* given by (14.18), (14.24) and (14.25) can be used to derive profiles of wind, temperature, and humidity in the surface layer in relation to momentum flux (τ), sensible heat flux (H), and water vapor flux (E). When sensible heat flux and evapotranspiration are positive, so that heat and water vapor are exchanged into the atmosphere, temperature and specific humidity decrease with height. This typically occurs during the day. At night, when sensible heat flux is negative (i.e., toward the surface), temperature increases with height.

The functions $\psi_m(\zeta)$, $\psi_h(\zeta)$, and $\psi_w(\zeta)$ account for the influence of atmospheric stability on turbulent fluxes. For unstable conditions ($\zeta < 0$),

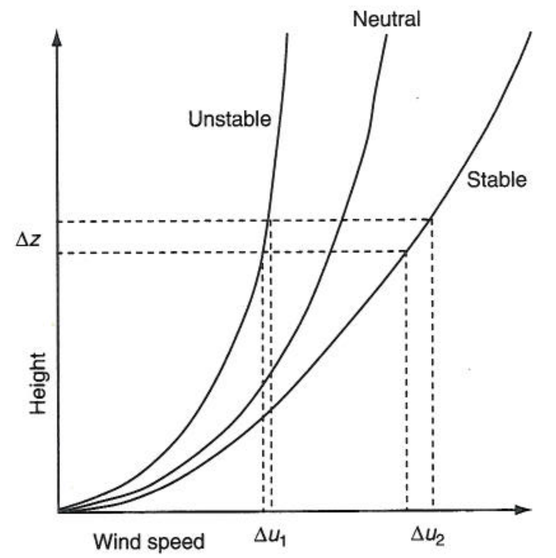

FIGURE 14.5. Effect of atmospheric stability on the vertical gradient of wind speed. A given vertical increment (Δz) produces a greater difference in wind speed in stable conditions (Δu_2) than in unstable conditions (Δu_1).

$$\begin{aligned} \psi_m(\zeta) &= 2\ln[(1+x)/2] + \ln\left[(1+x^2)/2\right] - 2\tan^{-1} x + \pi/2 \\ \psi_h(\zeta) &= \psi_w(\zeta) = 2\ln\left[(1+x^2)/2\right] \end{aligned} \quad (14.26)$$

where $x = (1 - 16\zeta)^{1/4}$. For a stable atmosphere ($\zeta \geq 0$),

$$\psi_m(\zeta) = \psi_h(\zeta) = \psi_w(\zeta) = -5\zeta \quad (14.27)$$

These functions have negative values when the atmosphere is stable and positive values when the atmosphere is unstable (Fig. 14.4). As a result, the vertical profile is weaker in an unstable surface layer, when turbulence efficiently mixes the air, than in stable conditions (Fig. 14.5).

Equations (14.17) and (14.18) can be combined to relate turbulent fluxes to their corresponding mean profile as

$$\begin{aligned} \tau &= \rho K_M(\partial\bar{u}/\partial z) \\ H &= -\rho C_p K_H(\partial\bar{\theta}/\partial z) \\ E &= -\rho K_W(\partial\bar{q}/\partial z) \end{aligned} \quad (14.28)$$

where

$$\begin{aligned} K_M &= ku_*(z-d)/\phi_m(\zeta) \\ K_H &= ku_*(z-d)/\phi_h(\zeta) \\ K_W &= ku_*(z-d)/\phi_w(\zeta) \end{aligned} \quad (14.29)$$

The terms K_M, K_H, and K_W are the eddy viscosity (K_M) and eddy diffusivity for heat (K_H) and water vapor (K_W). These coefficients increases with height (z) because of larger eddies (Fig. 14.1). To maintain a constant flux with respect to height, increases in transport with greater height must be balanced by corresponding decreases in the vertical gradient (e.g., $\partial\bar{u}/\partial z$). Indeed, observations show that in the surface layer mean vertical gradients of wind, temperature, and specific humidity decrease with height so that greater turbulent transfer is balanced by decreased vertical gradient and fluxes are constant (e.g., Fig. 14.3). Turbulent mixing also increases with greater wind speed, and the coefficients increase with greater wind speed. This is represented by the friction velocity (u_*). Greater turbulence results in a weaker vertical gradient. Enhanced mixing under unstable conditions is represented by the similarity functions $\phi_m(\zeta)$, $\phi_h(\zeta)$, and $\phi_w(\zeta)$. These functions have values <1 when the atmosphere is unstable and >1 when the atmosphere is stable so that K_M, K_H, and K_W increase as instability increases. To maintain constant flux, vertical gradients must decrease with increasing instability (Fig. 14.5).

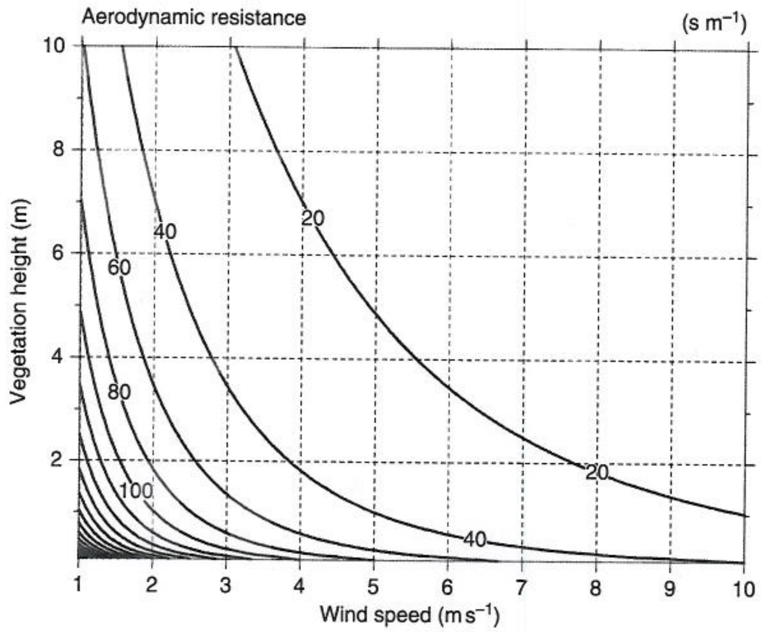

FIGURE 14.6. Aerodynamic resistance (r_{aM}) for neutral conditions in relation to vegetation height and wind speed. In this example, $z_{0M} = 0.1h$ and $d = 0.7h$, where h is vegetation height.

14.7 Bulk transfer equations

It is common in the atmospheric sciences to write the equations for momentum, sensible heat, and water vapor fluxes as the bulk transfer between the atmosphere at height z with variables $\bar{u}$, $\bar{\theta}$, and $\bar{q}$ and the surface with $\bar{u}_s = 0$, $\bar{\theta}_s$, and $\bar{q}_s$:

$$\begin{aligned} \tau &= \rho C_M \bar{u}(\bar{u} - \bar{u}_s) = \rho C_M \bar{u}^2 \\ H &= -\rho C_p C_H \bar{u}(\bar{\theta} - \bar{\theta}_s) \\ E &= -\rho C_W \bar{u}(\bar{q} - \bar{q}_s) \end{aligned} \tag{14.30}$$

The coefficient C_M is the dimensionless drag coefficient between the surface and atmosphere at height z. Similarly, the coefficients C_H and C_W are dimensionless heat and moisture transfer coefficients between the surface and atmosphere. These coefficients are

$$\begin{aligned} C_M &= k^2\left[\ln\left(\frac{z-d}{z_{0M}}\right) - \psi_m(\zeta)\right]^{-2} \\ C_H &= k^2\left[\ln\left(\frac{z-d}{z_{0M}}\right) - \psi_m(\zeta)\right]^{-1}\left[\ln\left(\frac{z-d}{z_{0H}}\right) - \psi_h(\zeta)\right]^{-1} \\ C_W &= k^2\left[\ln\left(\frac{z-d}{z_{0M}}\right) - \psi_m(\zeta)\right]^{-1}\left[\ln\left(\frac{z-d}{z_{0W}}\right) - \psi_w(\zeta)\right]^{-1} \end{aligned} \tag{14.31}$$

Typical values under neutral conditions are on the order of 10^{-3}. These coefficients increase with larger roughness length due to the greater turbulence generated by rougher surfaces. They decrease with increasing stability due to less turbulent motion and mixing.

The equations for momentum, sensible heat, and water vapor fluxes can also be written in a manner analogous to diffusion, proportional to a gradient times a conductance. Owing to the additive property of resistances when in series, it is more convenient to write the fluxes in a resistance notation:

$$\begin{aligned} \tau &= \rho(\bar{u} - \bar{u}_s)/r_{aM} = \rho\bar{u}/r_{aM} \\ H &= -\rho C_p(\bar{\theta} - \bar{\theta}_s)/r_{aH} \\ E &= -\rho(\bar{q} - \bar{q}_s)/r_{aW} \end{aligned} \tag{14.32}$$

The resistances r_{aM}, r_{aH}, and r_{aW} are aerodynamic resistances (s m^{-1}) to momentum, heat, and moisture, respectively, between the atmosphere at height z and the surface:

$$\begin{aligned} r_{aM} &= \frac{1}{k^2 u}\left[\ln\left(\frac{z-d}{z_{0M}}\right) - \psi_m(\zeta)\right]^2 \\ r_{aH} &= \frac{1}{k^2 u}\left[\ln\left(\frac{z-d}{z_{0M}}\right) - \psi_m(\zeta)\right]\left[\ln\left(\frac{z-d}{z_{0H}}\right) - \psi_h(\zeta)\right] \\ r_{aW} &= \frac{1}{k^2 u}\left[\ln\left(\frac{z-d}{z_{0M}}\right) - \psi_m(\zeta)\right]\left[\ln\left(\frac{z-d}{z_{0W}}\right) - \psi_w(\zeta)\right] \end{aligned} \tag{14.33}$$

where u is wind speed (m s^{-1}) at height z. Typical values for a neutral surface layer are in the range 10–100 s m^{-1}. Aerodynamic resistance decreases with increasing vegetation height due to greater roughness length (Fig. 14.6).

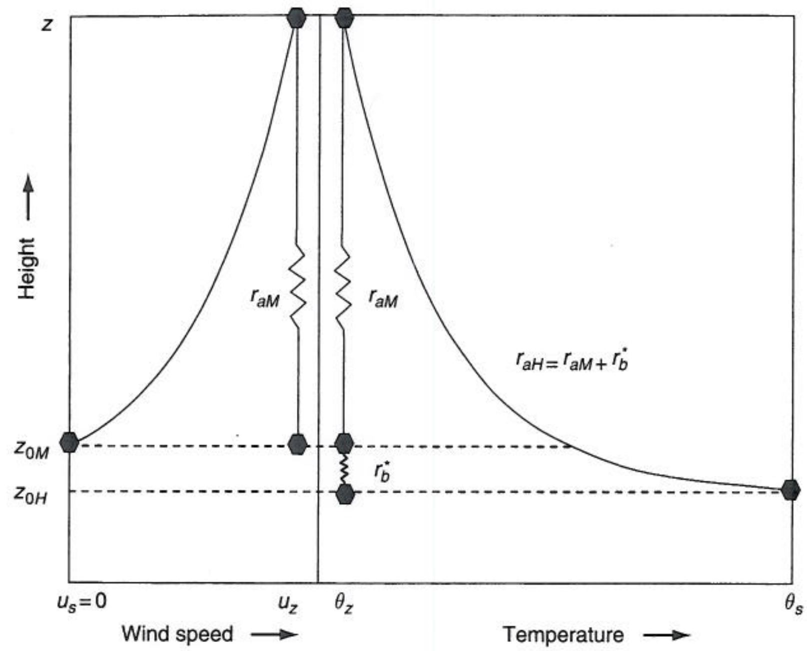

FIGURE 14.7. Schematic diagram of the aerodynamic resistances r_{aM} and r_{aH} and the excess resistance r_b^*. The resistance r_{aM} governs the wind profile from height z with wind u_z to the surface at z_{0M} with $u_s = 0$. The resistance r_{aH} governs the temperature profile from height z with temperature θ_z to the surface at z_{0H} with temperature θ_s. The excess resistance r_b^* is the resistance to heat transfer between heights z_{0M} and z_{0H}.

These resistances are less than the neutral value when the atmosphere is unstable, resulting in a large flux for a given vertical gradient, and more than the neutral value when the atmosphere is stable, resulting in a smaller flux for the same vertical gradient. Aerodynamic resistance decreases as roughness length increases. Aerodynamic resistances are related to drag coefficients as

$$\begin{aligned} r_{aM} &= (C_M u)^{-1} \\ r_{aH} &= (C_H u)^{-1} \\ r_{aW} &= (C_W u)^{-1} \end{aligned} \tag{14.34}$$

The roughness lengths for heat and moisture in a bulk aerodynamic formulation of vegetated surfaces are typically 10% that for momentum. That is, the apparent sources of heat and moisture in general occur at a lower height in the canopy than the apparent sink of momentum, and the aerodynamic resistances for sensible heat and water vapor are greater than that for momentum. The derivation of this excess resistance is most obvious under neutral conditions (Fig. 14.7). In this case, the aerodynamic resistance for heat is

$$\begin{aligned} r_{aH} &= \frac{\ln[(z-d)/z_{0H}]}{ku_*} = \frac{\ln[(z-d)/z_{0M}]}{ku_*} + \frac{\ln[z_{0M}/z_{0H}]}{ku_*} \\ &= r_{aM} + r_b^* \end{aligned} \tag{14.35}$$

The term r_b^* is the additional boundary layer resistance for sensible heat exchange from the canopy and accounts for the smaller roughness length for heat compared with momentum (Monteith and Unsworth 1990; Garratt, 1992).

14.8 Review questions

1. The covariance between potential temperature and vertical velocity measured by eddy covariance over a period of time is $0.2\,\mathrm{K\,m\,s^{-1}}$. What is the sensible heat flux? Assume $\rho = 1.15\,\mathrm{kg\,m^{-3}}$ and $C_p = 1005\,\mathrm{J\,kg^{-1}\,K^{-1}}$.
2. The wind speed measured at height $z = 25$ m is $5\,\mathrm{m\,s^{-1}}$. Calculate the wind speed at $z = 2$ m above a surface with $z_{0M} = 0.05$ m and $d = 0$. Ignore the effects of atmospheric stability. What is the momentum flux?
3. Calculate surface temperature given: surface sensible heat flux $H = 200\,\mathrm{W\,m^{-2}}$ and $\theta = 30$ °C and $u = 3\,\mathrm{m\,s^{-1}}$ at $z = 20$ m. The surface has the characteristics: $z_{0M} = 0.05$ m, $z_{0H} = 0.005$ m, and $d = 0.35$ m. Assume $\rho = 1.15\,\mathrm{kg\,m^{-3}}$, $C_p = 1005\,\mathrm{J\,kg^{-1}\,K^{-1}}$, $\psi_m(\zeta) = 0.8$, and $\psi_h(\zeta) = 1.4$.
4. Calculate the aerodynamic resistance r_{aH} between height $z = 25$ m where $u = 3\,\mathrm{m\,s^{-1}}$ and the surface for forest ($z_{0M} = 1.0$ m, $z_{0H} = 0.1$ m, and $d = 7$ m) and grass ($z_{0M} = 0.05$ m, $z_{0H} = 0.005$ m, and $d = 0.35$ m) for (a) $\zeta = 0$ (neutral), (b) $\zeta = -0.5$ (unstable), and (c) $\zeta = 0.5$ (stable).
5. If the temperature at $z = 25$ m is $\theta = 20$ °C and the surface sensible heat flux is $H = 175\,\mathrm{W\,m^{-2}}$, what is the surface temperature θ_s for the forest and grass

in problem 4(a)–(c)? Use $\rho = 1.15\ \text{kg m}^{-3}$ and $C_p = 1005\ \text{J kg}^{-1}\ \text{K}^{-1}$. Which has the higher surface temperature (forest or grass)? How does surface temperature vary with atmospheric stability? Why?

14.9 References

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