Surface Energy Budget

Please read Bonan Chapter 13

Energy Budget Concept

- For any system,

  (Energy in) – (Energy out) = (Change in energy)

- For the land surface,
  - Energy in = ?
  - Energy Out = ?
  - Change in energy = ?

Energy Budget Concept

- For the land surface,
  - Energy in = Radiation
  - Energy Out = Radiation + Turbulent fluxes of "sensible" and "latent" heat
  - Change in energy = changes in temperature of soil, plants, water, and air

Surface Radiation Budget

- Shortwave
  - Down (solar constant, seasonal and diurnal geometry, atmospheric attenuation, clouds and aerosol)
  - Up (albedo)
- Longwave
  - Down (emission from atmosphere depends on temperature profile, water vapor, clouds)
  - Up (surface temperature, emissivity)

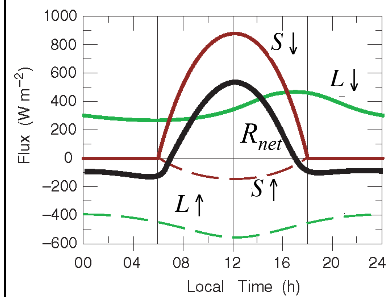
Surface Radiation Budget

$$R_{net} = SW\downarrow - SW\uparrow + LW\downarrow - LW\uparrow$$
$$= SW\downarrow(1-albedo) + LW\downarrow(1-\varepsilon) + \varepsilon\sigma T_s^{\,4}$$

- Shortwave
  - Down (solar constant * cosZ * transmissivity^{airmass})
  - Up (albedo * SW down)
- Longwave
  - Down (complicated! Weighted average of σT_a^4)
  - Up ($\varepsilon\, \sigma\, T_s^4$)

Radiation Budget Components

Diurnal Cycle



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- Net solar follows cos(*z*)
- LW fluxes much less variable ($\varepsilon\sigma T^4$)
  - LW up follows surface T as it warms through day
  - LW down changes little
  - LW net opposes SW_{net}
- R_{net} positive during day, slightly negative at night

Land Surface Energy Budget

- Very little of the energy gained by net radiation is stored in the ground (**G**)
- Most is emitted as LW IR and turbulent fluxes of sensible (**H**) and latent heat (**LE**)
- Latent energy is then **released into atmosphere** when vapor condenses

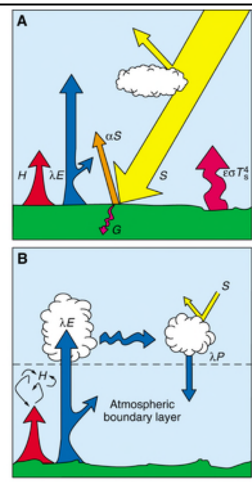


Fig. 1. Interactions between the land surface and the atmosphere that have direct impacts on the physical climate system. (**A**) Surface radiation budget. (**B**) Effect of heat fluxes on the atmosphere.

Surface Energy Budget

Storage change = Energy in – energy out

$$\rho c \frac{\Delta T}{\Delta t}\Delta z = (S\downarrow - S\uparrow + L\downarrow - L\uparrow) - H + \lambda E = G$$

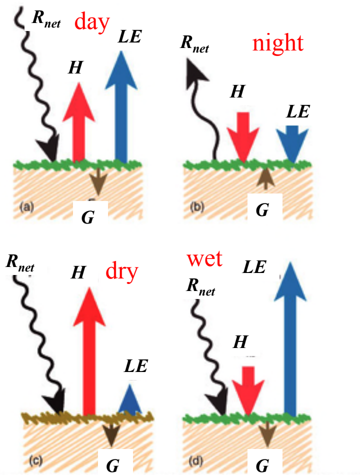
$$R_{net} = (S\downarrow - S\uparrow) + (L\downarrow - L\uparrow) = H + \lambda E + G$$

net SW *net LW* *sensible heat flux* *Latent heat of vaporization* *evap* *ground heat flux*

Role of the land surface:

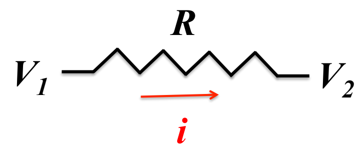
Partition of net radiation into turbulent fluxes & storage

Surface Energy Budgets



- R_{net} = H + LE + G
  ~ H + LE
- Daytime turbulent fluxes upward
- Night: turbulent fluxes downward (dew or frost!)
- Dry surfaces
  R_{net} ~ H
- Wet surfaces
  R_{net} ~ LE

Remember Ohm's Law?

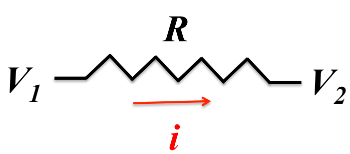


$$\underbrace{\Delta V}_{\text{voltage}} = ir$$

$$\underbrace{i}_{\text{current}} = \frac{\Delta V}{\underbrace{r}_{\text{resistance}}}$$

- Flow of current through a resistor is ratio of difference in potential to resistance (this is just a "*definition* of resistance")
- This is another form of our familiar concept of stuff flowing from high concentration to low concentration (like "Fickian Diffusion")

Heat Fluxes ~ Currents



$$\underbrace{\Delta V}_{\text{voltage}} = ir$$

$$\underbrace{i}_{\text{current}} = \frac{\Delta V}{\underbrace{r}_{\text{resistance}}}$$

- Sensible heat flux
  - Driving potential is a difference in temperature
  - H is proportional to ΔT
- Latent heat flux
  - Driving potential is a difference in vapor pressure
  - LE is proportional to Δe

Sensible Heat Flux

- Driving potential is a difference in temperature
- H is proportional to ΔT

$$\underbrace{H}_{\text{Watts m}^{-2}} = \underbrace{\rho}_{\substack{\text{density} \\ \text{kg m}^{-3}}} c_p \frac{\Delta T}{r} = \rho \underbrace{c_p}_{\substack{\text{heat capacity} \\ 1004\ \text{J K}^{-1}\ \text{kg}^{-1}}} \frac{T_s - T_a}{\underbrace{r}_{\substack{\text{aerodynamic} \\ \text{resistance} \\ \text{s m}^{-1}}}}$$

$$\frac{(\text{kg m}^{-3})(\text{J K}^{-1}\ \text{kg}^{-1})(\text{K})}{(\text{s m}^{-1})} = \frac{(\cancel{\text{kg}}\ \text{m}^{-3})(\text{J}\ \cancel{\text{K}}^{-1}\ \cancel{\text{kg}}^{-1})(\cancel{\text{K}})}{(\text{s m}^{-1})} = \frac{\text{J s}^{-1}}{\text{m}^2} = \text{W m}^{-2}$$

Water Vapor Pressure

- Molecules in an air parcel all contribute to pressure
- Each subset of molecules (e.g., N_2, O_2, H_2O) exerts a *partial* pressure
- The VAPOR PRESSURE (*e*), is the pressure exerted by water vapor molecules in the air

Latent Heat Flux

- Driving potential is a difference in water vapor pressure
- LE is proportional to Δe

$$LE = \frac{\rho c_p}{\gamma}\frac{\Delta e}{r} = \frac{\rho c_p}{\gamma}\frac{e_S - e_a}{r}$$

Watts m^{-2}

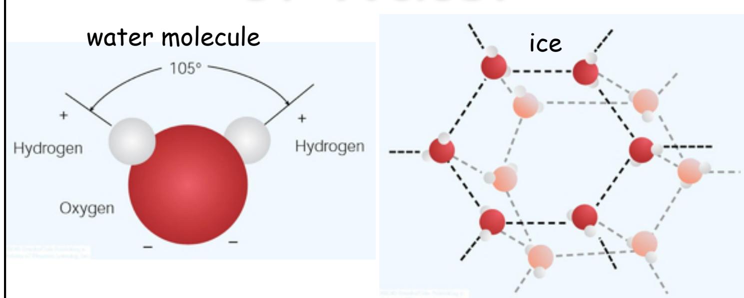
"Psychrometric constant"

$\gamma = (C_p P)/(0.622\lambda)$

66.5 Pa °C^{-1}

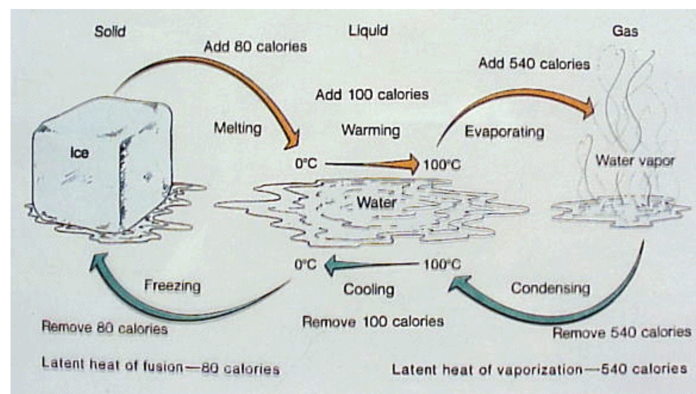
aerodynamic resistance s m^{-1}

Molecular Structure of Water



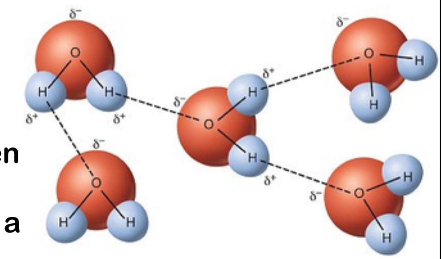
Water's unique molecular structure and hydrogen bonds enable all 3 phases to exist in Earth's environments

"Latent" (hidden) Energy associated with phase changes



Why does it take so much energy to evaporate water?

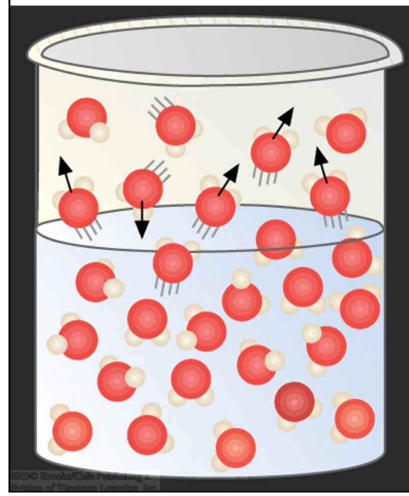
- In the liquid state, adjacent water molecules attract one another
- This same hydrogen bond accounts for surface tension on a free water surface



"plus" charge on hydrogen in one water molecule attracts the "minus" charge on a neighbor's oxygen

Evaporation must break these hydrogen bonds

Water Vapor Saturation



- Water molecules move between the liquid and gas phases
- When the rate of water molecules entering the liquid equals the rate leaving the liquid, we have equilibrium
- The air is said to be saturated with water vapor at this point

Clausius-Clapeyron Eqn

(see Monteith & Unsworth, pp 11-13)

- From Second Law of Thermodynamics

change in saturation vapor pressure — *Latent heat of vaporization*

$$\frac{de_s}{dT} = \frac{L}{T(\alpha_2 - \alpha_1)}$$

temperature — *specific volumes of liq & vapor*

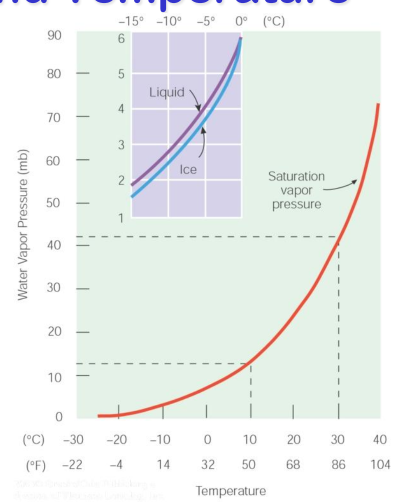
- Approximate but very accurate 0° to 35° C

$$e_s(T) = e_s(T^*)\exp\{A(T - T^*)/(T - T')\}$$

where $A = 17.27$, $T^* = 273$ K ($e_s(T^*) = 0.611$ kPa), and $T' = 36$ K.

Saturation and Temperature

- The saturation vapor pressure of water increases with temperature
  - At higher T, faster water molecules in liquid escape more frequently causing equilibrium water vapor concentration to rise
  - We sometimes say "*warmer air can hold more water*"
- There is also a vapor pressure of water over an ice surface
  - The saturation vapor pressure above solid ice is less than above liquid water



Latent Heat Flux

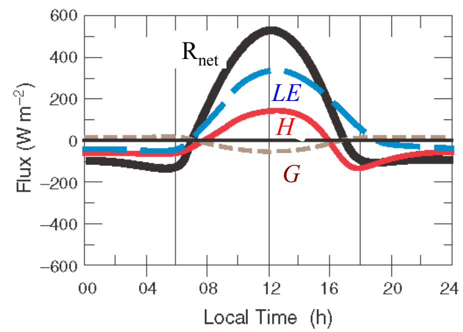
- Driven by difference in vapor pressure
- Over open water $e_{surface} = e_{sat}(T_s)$
- Over vegetation, liquid water is evaporating inside tiny openings in leaves called *stomata* (singular "*stomate*")
- Evapotranspiration = latent heat flux is driven by the *vapor pressure deficit*

$$\text{vpd} = (e_{sat}(T_s) - e_a)$$

$$LE = \frac{\rho c_p}{\gamma}\frac{e_s - e_a}{r} = \frac{\rho c_p}{\gamma}\frac{e_{sat}(T_s) - e_a}{r}$$

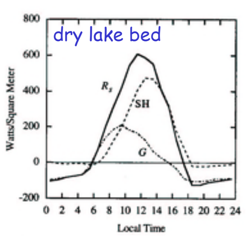
Idealized Diurnal Cycle

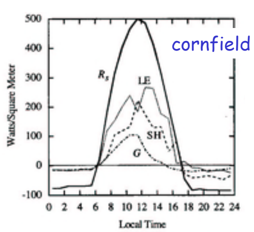
- R_{net} follows cos(z) during day, negative at night (LW cooling)
- Downward turbulent fluxes at night
- Ground heat flux smaller: downward durng day and up at night



R_{net} = H + LE + G
~ H + LE

Energy Budget Components

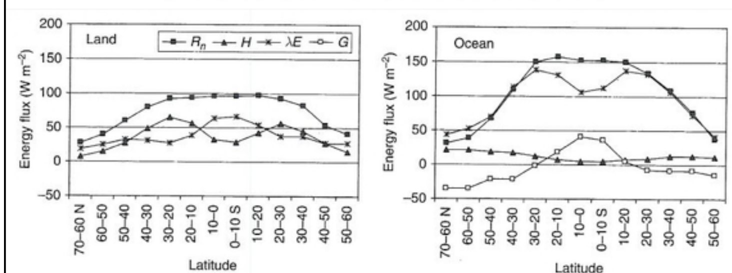




Observed Diurnal Cycles

- Recall R_{net} = H + LE + G
- R_{net} follows cos(z) during day
- R_s < 0 at night due to LW cooling at surface
- Over desert, LE = 0 (dry), so R_{net} balanced early by G, later by H
- Over active vegetation, R_{net} balanced by all three terms
  - H+LE >> G
  - Note dip in LE at mid-day … why?

Global Variations



- Tropics -> pole gradients
- Lower albedo over oceans (higher R_{net})
- G ~ 0 : Storage negligible over land
- Water budget in oceans and required atmospheric transport

Radiation, Hydrology, & the Sfc Energy Budget

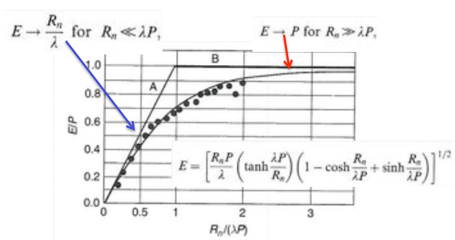
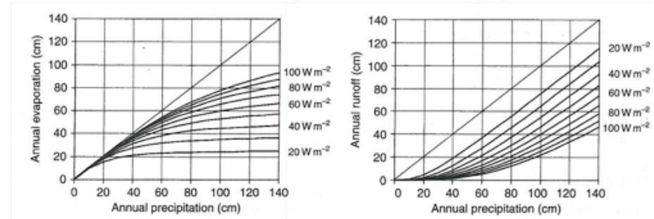


FIGURE 13.2. Relationship between the ratio of annual evaporation to precipitation (E/P) and the ratio of net radiation to the amount of energy required to evaporate the annual precipitation ($R_n/\lambda P$). Redrawn from Budyko (1974, p. 325).

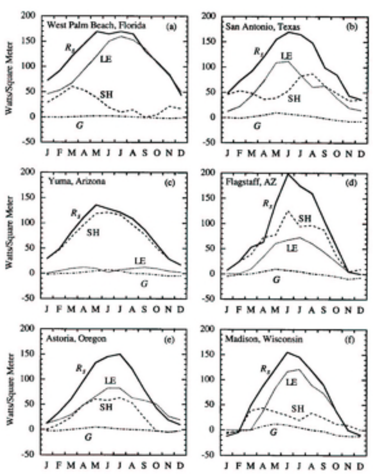
- Wet places: evaporation balances radiation
- Dry places: evaporation balances precipitation
- Smooth transition in between

Annual Radiation and Hydrology



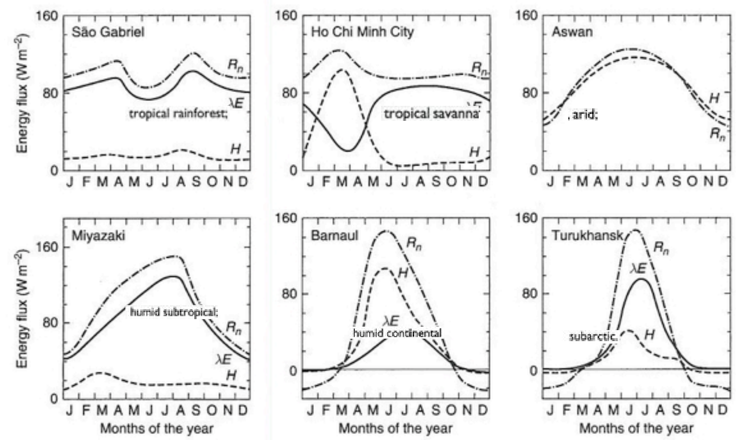
- Radiation favors evaporation over runoff
- Dry (sunny) places: LE ~ Precipitation
- Wet (cloudy) places: Runoff ~ Precipitation

Seasonal Energy Budgets

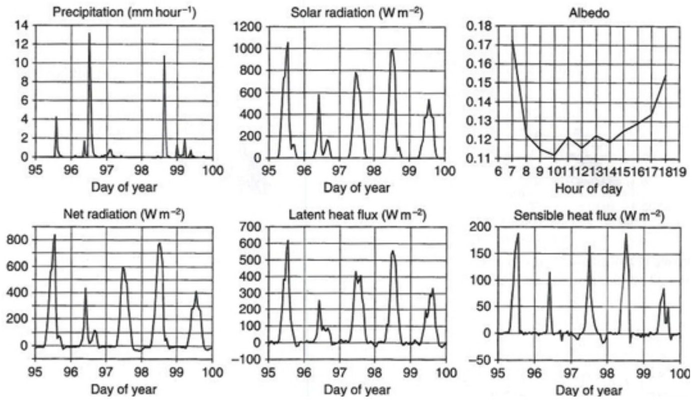


- Seasonal course of R_{net} due to Sun-Earth geometry
- Moist climates feature near balance of R_{net} ~ LE
- Dry climates feature near balance of R_{net} ~ H
- Others are intermediate
  - Spring vs fall in Texas
  - Summer (leaves) vs spring and fall in Wisc
- (H, LE) >> G everywhere

Seasonal Energy Fluxes

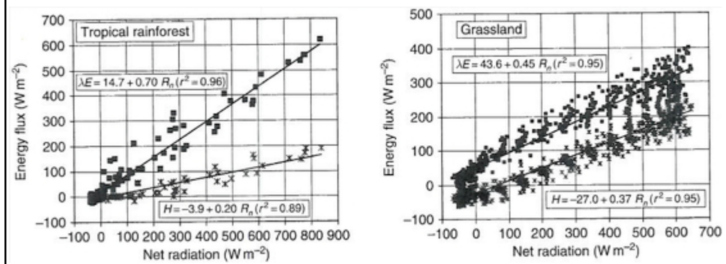


Diurnal Variations



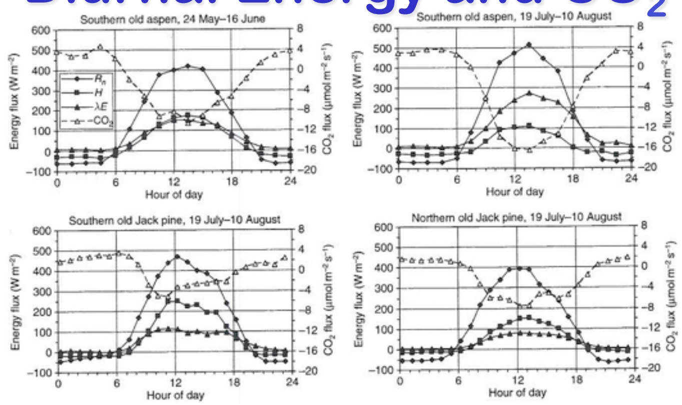
Tropical Forest: Rondonia, Brazil (10° S)

Partition of Net Radiation



- Energy budget "closure"
- Forest "harvests" radiation to extract water from soil
- Grassland passes more energy back to atmosphere as sensible heat

Diurnal Energy and CO_2



- CO_2 flux is a mirror image of LE (stomatal control)
- Physiological differences: broadleaf vs needleleaf

Partition of Net Radiation

Ground heat flux

$G = k(T_s - T_g)/\Delta z$

$$R_{net} = (S\downarrow - S\uparrow) + (L\downarrow - L\uparrow) = H + \lambda E + G$$

$$H = -\rho C_p \frac{(T_a - T_s)}{r_H}$$

Sensible flux driven by ΔT

Latent flux driven by VPD

$$\lambda E = -\frac{\rho C_p}{\gamma} \frac{(e_a - e_*[T_s])}{r_W}$$

$\gamma = (C_p P)/(0.622\lambda)$ 66.5 Pa °C^{-1}

"Psychrometric constant"

Surface Energy Budget

Energy in = energy out + storage change

$$(1-r)S\downarrow + \varepsilon L\downarrow = \varepsilon\sigma(T_s + 273.15)^4 + H + \lambda E + G$$

$$(1-r)S\downarrow + \varepsilon L\downarrow = \varepsilon\sigma(T_s + 273.15)^4 - \rho C_p \frac{(T_a - T_s)}{r_H} - \frac{\rho C_p}{\gamma}\frac{(e_a - e_*[T_s])}{r_W} + k\frac{(T_s - T_g)}{\Delta z}$$

- Can "solve" for surface temperature
- Physical properties: albedo, emissivity, heat capacity, soil conductivity & temperature
- "Resistances" are properties of the turbulence … depend sensitively on H!

Clausius-Clapeyron Eqn

(see Monteith & Unsworth, pp 11-13)

- Approximate but very accurate 0° to 35° C

$$e_s(T) = e_s(T^*)\exp\{A(T - T^*)/(T - T')\}$$

where $A = 17.27$, $T^* = 273$ K ($e_s(T^*) = 0.611$ kPa), and $T' = 36$ K.

- *Slope* $s = \Delta(e_s)/\Delta T$ accurate from 0° to 40° C

$$s = \lambda M_W e_s(T)/(RT^2)$$

λ = L = 2.48 J kg^{-1} M_W (mol wt water) = 0.018 mol kg^{-1}

R = 8.314 J mol^{-1} K^{-1} ("universal gas constant")

Penman-Monteith Equation

"Thermodynamic" energy balance

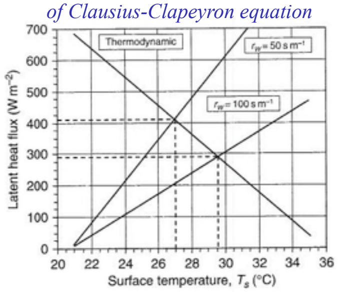
$$\lambda E = (R_n - G) - H = (R_n - G) + \rho C_p(T_a - T_s)/r_H$$

$$e_*[T_s] = e_*[T_a] + s(T_s - T_a)$$

"Turbulent" energy balance

$$\lambda E = \frac{\rho C_p}{\gamma}\frac{(e_*[T_a] + s(T_s - T_a) - e_a)}{r_W}$$

VPD approximated by linearization of Clausius-Clapeyron equation



Solve for surface temperature

$$T_s - T_a = (r_H/\rho C_p)(R_n - G - \lambda E)$$

Solutions to P-M Equation

Latent heat flux

$$\lambda E = \frac{s(R_n - G) + \rho C_p(e_*[T_a] - e_a)/r_H}{s + \gamma(r_W/r_H)}$$

Sensible heat flux

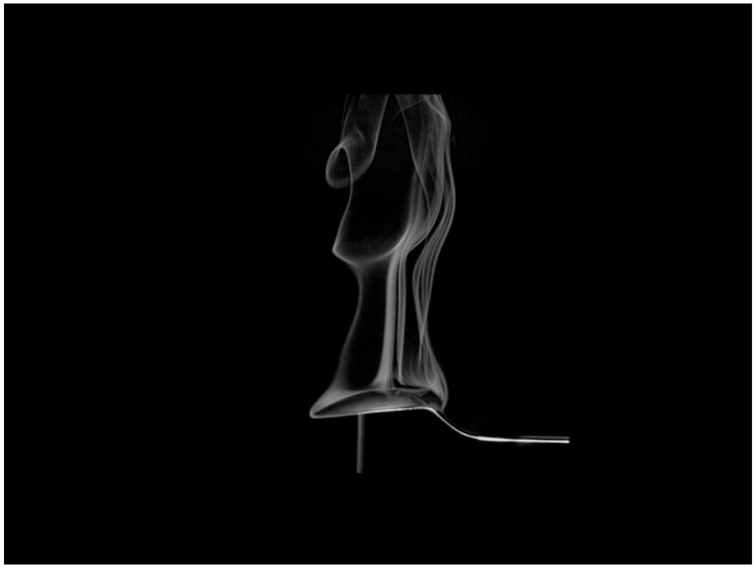
$$H = \frac{(R_n - G)\gamma^* - \rho C_p(e_*[T_a] - e_a)/r_H}{s + \gamma^*}$$

Surface temperature

$$T_s = T_a + \frac{(R_n - G)\gamma^* r_H/\rho C_p - (e_*[T_a] - e_a)}{s + \gamma^*}$$

Turbulent Fluxes

Please read Bonan, Chapter 14



Richardson's Rhyme

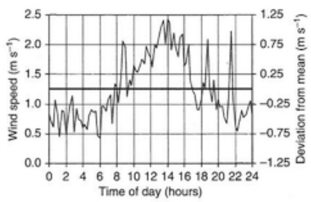
- "Big whorls have little whorls,
  Which feed on their velocity;
  And little whorls have lesser whorls,
  And so on to viscosity"
  - Lewis Richardson, *The supply of energy from and to Atmospheric Eddies* 1920

- "Great fleas have little fleas
  Upon their backs to bite 'em,
  And little fleas have lesser fleas,
  And so, ad infinitum"
  - Augustus De Morgan
    (19th century mathematician, parodying Jonathon Swift, 1733)

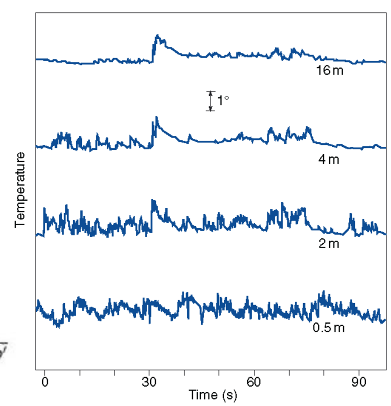
Sonic Anemometer

- Measures elapsed time for sound pulses to cross air in 3D
- Speed of sound is a known function of temperature
- Relative motion determined accurately in 3D
- Very fast instrument response time

Time Series of Turbulence



$$a = \bar{a} + a'$$
$$a' = a - \bar{a}.$$



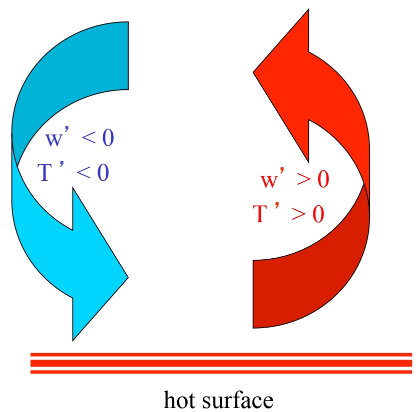
$$\bar{a} = \overline{(\bar{a} + a')} = \overline{(\bar{a})} + \overline{a'} = \bar{a} + \overline{a'}$$
$$\sigma^2 = \frac{1}{N}\Sigma(a_i - \bar{a})^2 = \overline{a'a'}$$

$$\overline{ab} = \bar{a}\bar{b} + \overline{a'b'}$$

$$\mathrm{cov}(a,b) = \frac{1}{N}\Sigma(a_i - \bar{a})(b_i - \bar{b}) = \overline{a'b'}$$

Turbulent Heat Flux

$$w \equiv \overline{w} + w' \qquad T \equiv \overline{T} + T'$$



- Imagine a turbulent eddy over a hot surface
- Updrafts are systematically warmer than downdrafts

- Updraft:
  $\overline{w'T'} > 0$
- Downdraft:
  $\overline{w'T'} > 0$

Sensible Heat Flux (Reynolds' Averaging)

Upward sensible heat flux $H = c_p \rho wT$

Heat capacity at constant pressure 1004 J K^{-1} kg^{-1}

density of air p/RT (~ 1.2 kg m^{-3})

Vertical velocity (m s^{-1})

Temp (K)

$$w = \overline{w} + w', \qquad T = \overline{T} + T'$$

Total; time mean; turbulent fluctuation

$$\overline{wT} = \overline{(\overline{w}+w')(\overline{T}+T')}$$
$$= \overline{\overline{w}\,\overline{T} + \overline{w}T' + w'\overline{T} + w'T'}$$
$$= \overline{\overline{w}\,\overline{T}} + \overline{\overline{w}T'} + \overline{w'\overline{T}} + \overline{w'T'}$$
$$= \overline{w}\,\overline{T} + \cancel{\overline{w}\,\overline{T'}}^{0} + \cancel{\overline{w'}\,\overline{T}}^{0} + \overline{w'T'}$$

$$\boxed{\overline{wT} = \overline{w}\,\overline{T} + \overline{w'T'}}$$

- Mean of a mean is the mean
- Mean of a prime is zero
- Mean of a product is **not** necessarily zero

Variance, Covariance, Correlation

$\overline{c} \equiv \frac{1}{N}\sum_{i=1}^{N} c_i$ mean

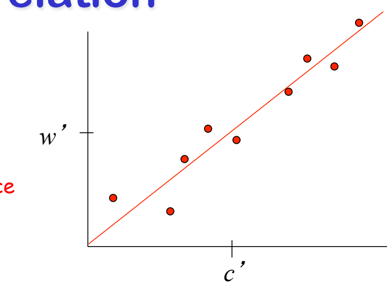
$c_i' \equiv c_i - \overline{c}$ perturbation

$\overline{c_i' c_i'} = \frac{1}{N}\sum_{i=1}^{N}(c_i - \overline{c})^2 \equiv \sigma_c^2$ variance

$\overline{w'^2} \equiv \sigma_w^2$ variance of w

$\overline{w'c'} = \frac{1}{N}\sum_{i=1}^{N}(c_i - \overline{c})(w_i - \overline{w}) \equiv \mathrm{cov}(w,c)$ covariance of w and c

$\frac{\overline{w'c'}}{\sigma_w \sigma_c} = r$ Normalized covariance is the correlation coefficient



Turbulent Fluxes

Sensible Heat Flux

$$\overline{wT} = \overline{w}\,\overline{T} + \overline{w'T'}$$

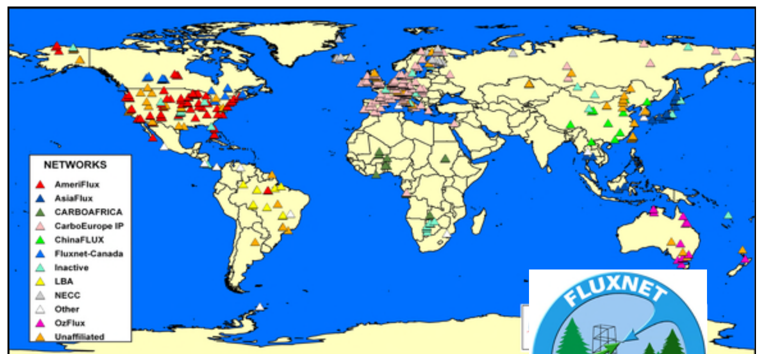
$$\text{Total} = \text{mean} + \text{eddy}$$

$$\overline{wq} = \overline{w}\,\overline{q} + \overline{w'q'}$$

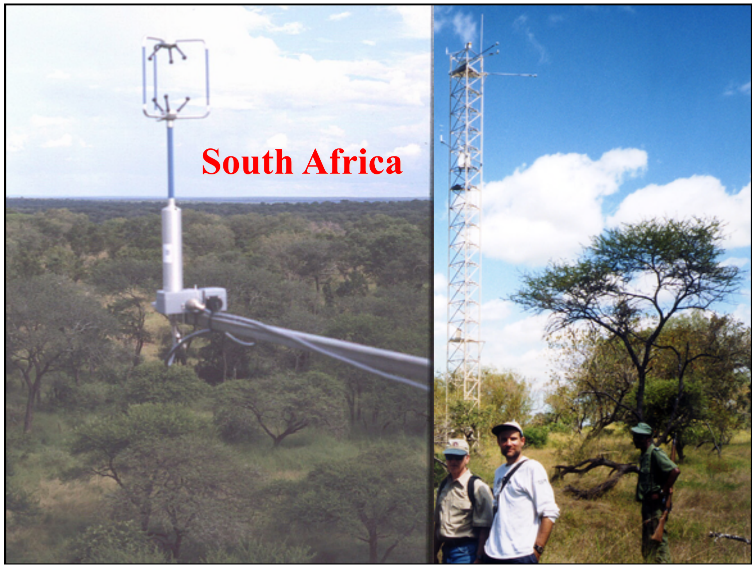
Latent Heat Flux

- Near the ground, $\overline{w} \approx 0$
- "Eddy" terms dominate
- How can these fluxes be **measured**?

Weird units: K m s^{-1}; kg kg^{-1} m s^{-1}



- **FluxNet**: An international "network of networks" > 550 sites
- 10 Hz measurements from many sites for **> 5 yrs**
- **H, LE, and NEE** of CO_2 at most sites
- Data available online: http://fluxnet.ornl.gov

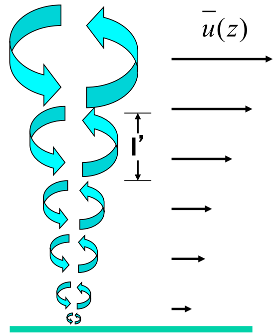

South Africa


Boreal Forest


WLEF-TV Tower
Rhinelander, WI
450 m tall
10 Hz measurements since 1995
3 levels


Tropical Forest
See http://www-as.harvard.edu/chemistry/brazil/tower_diagram.html

Surface-Layer Mixing



- Turbulent eddies near the surface act to mix atmospheric properties (T, q, u) and reduce vertical gradients
- Assume a characteristic length scale l' for eddy mixing, then

$$u = \bar{u}(z) + u'$$

$$u' = -l' \frac{\partial \bar{u}}{\partial z}$$

If eddies are isotropic (length and depth similar), then

$$|w'| \sim |u'|, \quad \text{so} \quad w' \sim l' \frac{\partial \bar{u}}{\partial z}$$

Surface Layer Stress

$$\tau_x \equiv -\rho \overline{w'u'}$$

$$= -\rho \overline{\left(-l' \frac{\partial \bar{u}}{\partial z}\right)\left(-l' \frac{\partial \bar{u}}{\partial z}\right)} = -\rho \overline{l'^2} \left|\frac{\partial \bar{u}}{\partial z}\right|^2$$

$$= K_m \frac{\partial \bar{u}}{\partial z}, \quad \text{where } K_m \equiv \rho \overline{l'^2} \left|\frac{\partial \bar{u}}{\partial z}\right|$$

$$\text{Define } u_* \equiv \sqrt{\frac{\tau_x}{\rho}} = \left(\overline{u'w'}\right)^{1/2} \quad \text{then}$$

$$\frac{\tau_x}{\rho} = \frac{K_m}{\rho} \frac{\partial \bar{u}}{\partial z} = u_*^2$$

- Momentum flux (surface stress) is proportional to the square of the product of the wind speed gradient (shear) and the turbulent length scale
- Define an "eddy viscosity" or "eddy diffusivity" K_m which is analogous to molecular diffusivity
- Define a velocity scale u_* for the turbulent eddies near the surface, called the friction velocity

Surface Layer (cont'd)

$$\frac{\tau_x}{\rho} = \frac{K_m}{\rho} \frac{\partial \bar{u}}{\partial z} = u_*^2$$

$$K_m = \rho \overline{l'^2} \left|\frac{\partial \bar{u}}{\partial z}\right| = \rho k^2 z^2 \left|\frac{\partial \bar{u}}{\partial z}\right|$$

$$\frac{K_m}{\rho} \frac{\partial \bar{u}}{\partial z} = u_*^2 = \frac{\rho k^2 z^2 \left|\frac{\partial \bar{u}}{\partial z}\right|}{\rho} \frac{\partial \bar{u}}{\partial z}$$

$$\left(\frac{\partial \bar{u}}{\partial z}\right)^2 = \frac{u_*^2}{k^2 z^2}$$

$$\frac{\partial \bar{u}}{\partial z} = \frac{u_*}{kz}$$

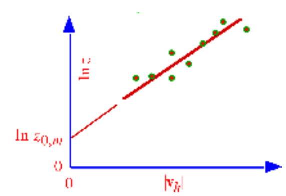
- Near the surface, eddies are limited in size by the proximity of the ground, so l' in K_m is $l'(z)$
- Assume $l' = kz$, where $k \sim 0.4$ is an empirical coefficient known as "von Karman's constant"
- Leads to a characteristic relationship for variation of mean wind speed with height: the log-wind profile

Log Wind Profile

$$\frac{\partial \bar{u}}{\partial z} = \frac{u_*}{kz} \quad \text{separate variables}$$

$$\int_{z_0}^{z} \partial \bar{u} = \frac{u_*}{k} \int_{z_0}^{z} \frac{\partial z}{z}$$

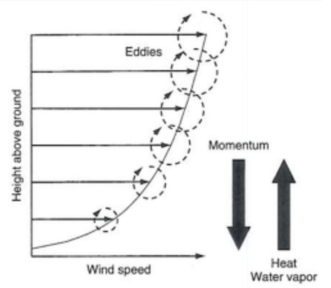
$$\bar{u}(z) = \frac{u_*}{k} \ln \frac{z}{z_0}$$



- Mean wind speed in the surface layer is decelerated by friction whose influence is felt aloft through eddy momentum flux
- Varies logarithmically with height
- Y-intercept of log-linear plot of SL wind vs z is z_0, which we define as the "roughness length"

Roughness Length

y-intercept of log-linear plot of SL wind vs z is z_0, which we define as the "roughness length"



Surface	Roughness length (m)
Soil	0.001–0.01
Grass	
Short	0.003–0.01
Tall	0.04–0.10
Crop	0.04–0.20
Forest	1.0–6.0
Suburban	
Low density	0.4–1.2
High density	0.8–1.8
Urban	
Short building	1.5–2.5
Tall building	2.5–10

Monin-Obukhov Similarity

For neutral conditions

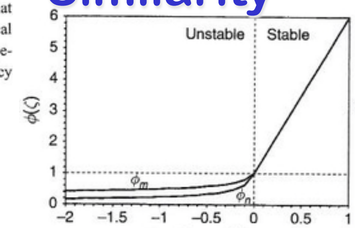
$(z/u_*)\partial\bar{u}/\partial z = 1/k$

The Monin–Obukhov similarity theory states that when scaled appropriately the dimensionless mean vertical gradients of wind ($\bar{u}$), potential temperature ($\bar{\theta}$), and specific humidity ($\bar{q}$) are unique functions of a buoyancy parameter (ζ):

$$\left[\frac{k(z-d)}{u_*}\right]\frac{\partial\bar{u}}{\partial z} = \phi_m(\zeta)$$

$$\left[\frac{k(z-d)}{\theta_*}\right]\frac{\partial\bar{\theta}}{\partial z} = \phi_h(\zeta)$$

$$\left[\frac{k(z-d)}{q_*}\right]\frac{\partial\bar{q}}{\partial z} = \phi_w(\zeta)$$



$$\zeta = (z-d)/L$$

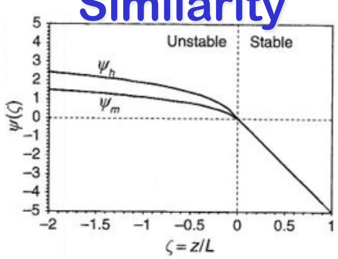
$$L = -u_*^3/[k(g/\theta)(H/\rho C_p)]$$

$$\phi_m^2(\zeta) = \phi_h(\zeta) = \phi_w(\zeta) = (1-16\zeta)^{-1/2} \quad \text{for } \zeta < 0 \text{ (unstable)}$$

$$\phi_m(\zeta) = \phi_h(\zeta) = \phi_w(\zeta) = 1 + 5\zeta \quad \text{for } \zeta \geq 0 \text{ (stable)}$$

- **Empirical adjustment of log-wind profiles to account for buoyancy fluxes (anisotropy)**

Surface-Layer Similarity



unstable conditions ($\zeta < 0$),

$$\psi_m(\zeta) = 2\ln[(1+x)/2] + \ln[(1+x^2)/2] - 2\tan^{-1}x + \pi/2$$

$$\psi_h(\zeta) = \psi_w(\zeta) = 2\ln[(1+x^2)/2]$$

where $x = (1-16\zeta)^{1/4}$

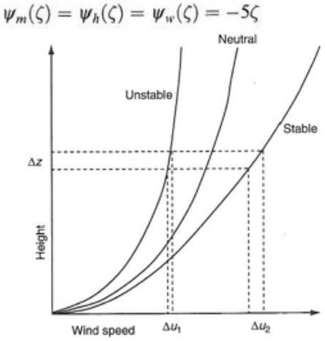
For a stable atmosphere ($\zeta \geq 0$),

$$\psi_m(\zeta) = \psi_h(\zeta) = \psi_w(\zeta) = -5\zeta$$

$$\bar{u}(z) = \frac{u_*}{k}\left[\ln\left(\frac{z-d}{z_{0M}}\right) - \psi_m(\zeta)\right]$$

$$\bar{\theta}(z) - \bar{\theta}_s = \frac{\theta_*}{k}\left[\ln\left(\frac{z-d}{z_{0H}}\right) - \psi_h(\zeta)\right]$$

$$\bar{q}(z) - \bar{q}_s = \frac{q_*}{k}\left[\ln\left(\frac{z-d}{z_{0W}}\right) - \psi_w(\zeta)\right]$$



Generalized (empirical) vertical profiles

Estimation of Turbulent Fluxes

$$\tau = \rho K_M(\partial\bar{u}/\partial z)$$

$$H = -\rho C_p K_H(\partial\bar{\theta}/\partial z)$$

$$E = -\rho K_W(\partial\bar{q}/\partial z)$$

where

$$K_M = ku_*(z-d)/\phi_m(\zeta)$$

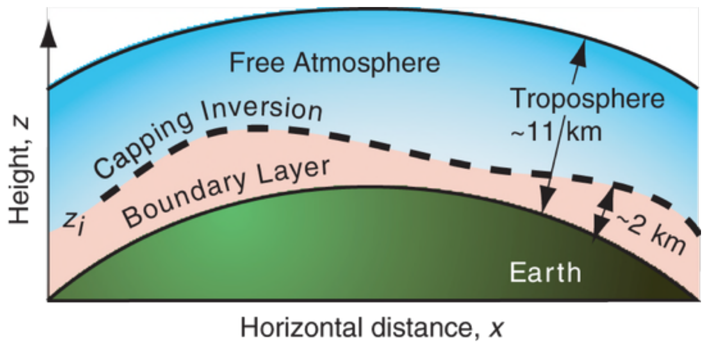
$$K_H = ku_*(z-d)/\phi_h(\zeta)$$

$$K_W = ku_*(z-d)/\phi_w(\zeta)$$

- **Fluxes are driven by gradients in u, *T*, and *q***
- **Fluxes are proportional to friction velocity**
- **These are simply definitions of K_M, K_H, K_W**
- **Ohm's Law combined with Similarity**

The Atmospheric Boundary Layer

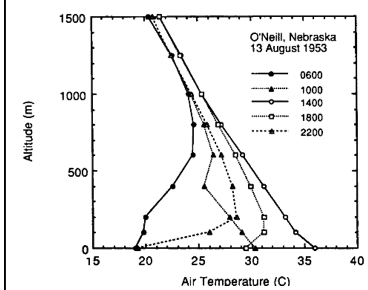
(a.k.a ABL, PBL, CBL, NBL, SBL …)



"The layer of atmosphere in turbulent contact with the surface"

PBL Temperatures

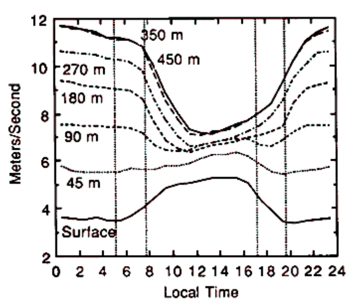
Diurnal Cycle



- Morning inversion broken by surface heating
- Shallow ML by 10 AM under RL from yesterday
- Superadiabatic surface layer at 2 PM
- New inversion forms near surface by 6 PM
- Nocturnal BL grows "from the bottom up"

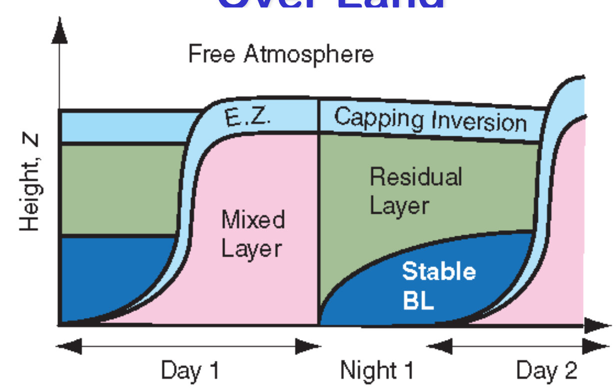
PBL Wind Speeds

Annual Mean Diurnal Cycles



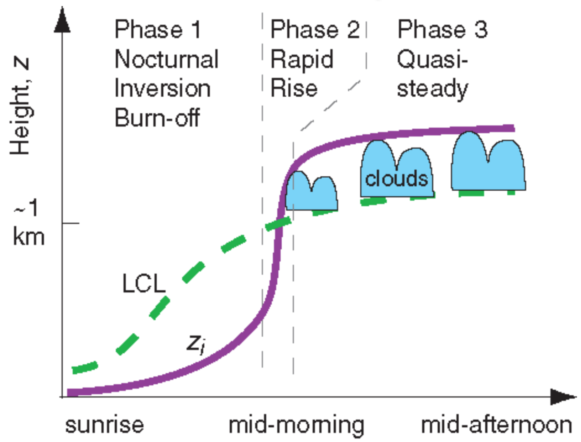
- Surface winds are maximum at midday
- Winds aloft are maximum at night (decoupling)
- Momentum mixing during daytime allows surface friction to be "felt" throughout ML

Typical Diurnal Cycle of PBL Over Land

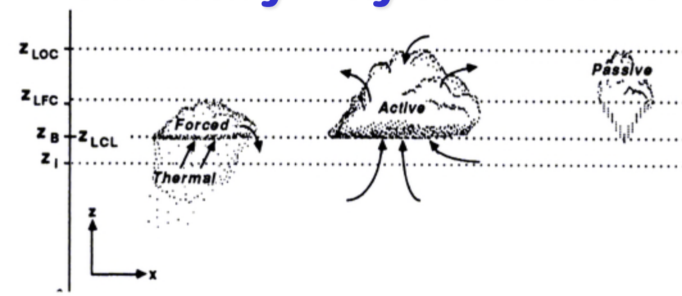


(Stull, 1988)

CBL Development



Boundary Layer Clouds



- Big thermals that reach lifting condensation level are often capped by shallow cumulus clouds
- If these clouds are forced to the level of free convection, they grow on their own by condensation heating
- PBL-top clouds are an important means for venting PBL air into the free troposphere